Analysis of the modified Verhulst equation

The modified Verhulst equation reads

$$\dot{x} = rx\left(\frac{g}{r}\frac{x}{x+c} - \frac{d}{r} - \frac{x}{K}\right) = f(x). \tag{1}$$

Here, g is the growth rate constant, d the death rate constant and $r = g - d$. K denotes the capacity of the environment and c is a parameter governing the population behaviour at low densities.
Stationary states are defined by $\dot{x} = 0$, implying

$$x = 0 \quad \text{or} \quad \frac{g}{r}\frac{x}{x+c} - \frac{d}{r} - \frac{x}{K} = 0. \tag{2}$$

The right equation is a quadratic with the solutions

$$x = \frac{1}{2}(K-c) \pm \sqrt{\frac{1}{4}(K-c)^2 - \frac{dcK}{r}}. \tag{3}$$

This equation has two real solutions if $\frac{1}{4}(K-c)^2 > \frac{dcK}{r}$, and no solution if $c > c_{\text{crit}}$ defined by the equation

$$\frac{1}{4}(K - c_{\text{crit}})^2 = \frac{dc_{\text{crit}}K}{r}. \tag{4}$$

The following plots allow to assess the stability of the three stationary points.