



Energy Metabolism and Thermodynamics

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Mathematical modelling





Bioinformatics





0.01

0.02

Average Shannon Entropy (H)

0.03

0.04

C1-metabolism - -C PS - -C 0.00

Thermodynamics

Science concerned with <u>heat</u> and its relation to <u>energy</u> and <u>work</u>

Developed out of a desire to increase the efficiency of steam engines

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wikipedia.org

Efficiency:

energy output of mechanical work that the engine produces energy input to the engine by the burning fuel

Thermodynamics in Biology?

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What is the output?

For unicellular organisms (E. coli): growth





www.libretexts.org

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Internal energy (as opposed to kinetic, potential energy etc.)

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Entropy

3: As a system approaches absolute zero the entropy of the system approaches a minimum value

Important for small absolute T, not relevant for biology

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The arrow of time!



[medium.com]

[streetsmash.com]

WHY?

Low entropy

High entropy

Gas: N particles, can be either left or right

N=10

left	right	Ω
0	10	1
1	9	10
2	8	45
3	7	120
4	6	210
5	5	252
6	4	210
7	3	120
8	2	45
9	1	10
10	0	1





 Ω : # microstates for macrostate

The famous Boltzmann formula:

 $S = k_B \ln(\Omega)$

For large numbers this becomes:

$S = -k_B \sum_i p_i \ln(p_i)$			
left	right	Ω	
0	10	1	
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Entropy



2nd law: The entropy of the universe is always increasing! The universe will reach equilibrium ('heat death')

How can something like life, with all its patterns and structure, exist if entropy always increases?

Consume food? (one atom is as good as any other; one calorie is as good as any other)

Export entropy!!

WHAT IS LIFE?

The Physical Aspect of the Living Cell

BY ERWIN SCHRÖDINGER SENIOR PROFESSOR AT THE DUBLIN INSTITUTE FOR ADVANCED STUDIES

> Based on Lectures delivered under the auspices of the Institute at Trinity College, Dublin, in February 1943

CAMBRIDGE AT THE UNIVERSITY PRESS 1944

Entropy (classical)

Entropy is defined as $\Delta S = \frac{\Delta Q}{T}$



Amount of heat added to a system divided by its temperature



1st law (energy conservation): $W = Q_H - Q_a$ Entropy change: $S_i = -\frac{Q_H}{T_H} + \frac{Q_a}{T} \ge 0$ (2nd law) Work done: $W = \left(1 - \frac{T_a}{T_H}\right) Q_H - T_a S_i$ $= W_{\text{max}} - (T_a S_i)$

Energy lost due to entropy production!

"heat engine"

isolated (no exchange of matter) adiabatic (no exchange of heat) **not closed!** (exchange of work!)



Internally produced entropy (>0) heat exchange metabolite exchange

- Entropy is always produced (metabolism, maintenance, ...)
- This needs to be balanced by exporting entropy (e.g. as heat or as low entropy molecular products)

Thermodynamic potentials

Internal energy: $\Delta U = T \Delta S - p \Delta V$ S and V constant: U decreases and reaches a minimum at equilibrium capacity to do work plus the capacity to release heat

(Helmholtz) Free energy: $F = U - TS \Rightarrow \Delta F = -S \Delta T - p \Delta V$ *T* and *V* constant: *F* decreases and reaches a minimum at equilibrium capacity to do mechanical work

Enthalpy: $H = U + pV \Rightarrow \Delta H = T \Delta S + V \Delta p$

S and *p* constant: *H* decreases and reaches a minimum at equilibrium capacity to do non-mechanical work plus the capacity to release heat

Gibbs free energy: $G=U+pV-TS \Rightarrow \Delta H=-S\Delta T+V\Delta p$ *T* and *p* constant: *G* decreases and reaches a minimum at equilibrium capacity to do non-mechanical work

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Constant *T* and *p* are relevant for biochemical processes!!



Chemical reactions

A **≓** B

We can measure concentrations: $[A]_{obs}$, $[B]_{obs}$

and in equilibrium:

 $[A]_{eq}, [B]_{eq}$

Equilibrium constant:

$$K_{\rm eq} = \frac{[\rm B]_{\rm eq}}{[\rm A]_{\rm eq}}$$

Mass-action ratio: Γ

$$= \frac{[B]_{obs}}{[A]_{obs}}$$

In equilibrium: $\Gamma = K_{eq}$

$$\Delta G = -RT \ln \left(K_{\rm eq} / \Gamma \right)$$

If $\Gamma > K_{eq}$: $\Delta G > 0$ reaction proceeds from B to A If $\Gamma < K_{eq}$: $\Delta G < 0$ reaction proceeds from A to B

$A \rightleftharpoons B$

What is ΔG^0 ?

Change in Gibbs free energy if all reactants are present in standard concentrations (1M)

 $\Rightarrow \Gamma = 1M/1M = 1$

$$\Rightarrow \Delta G^{0} = -RT \ln (K_{eq}/\Gamma) = -RT \ln K_{eq}$$

Dihydroxyacetone phosphate \Rightarrow glyceraldehyde-3-phosphate DHAP \Rightarrow GAP

Experimental: $\Delta G^0 = +1.82 \text{ kcal/mol} = +7.61 \text{ kJ/mol}$

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If TPI is in equilibrium, the concentration of DHAP is 22 times higher than of GAP

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The numerical values of Γ and K_{eq} depend on the measurement system!

Aldolase: FBP \rightleftharpoons GAP + DHAP $K_{eq} = \frac{[GAP]_{eq} \cdot [DHAP]_{eq}}{[FBP]_{eq}}$

Here: $\Delta G^0 = 23.8 \text{ kJ/mol}$

Thus: $K_{\rm eq}/1M = \exp(-\Delta G^0/RT) = 6.7 \cdot 10^{-5}$



What's going wrong?



If
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But who cares what happens at 1M?



What's going wrong? This only applies to standard concentrations (1M):

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But who cares what happens at 1M?

If $[GAP]_{eq} = 50 \,\mu$ M and $[DHAP]_{eq} = 50 \,\mu$ M then $[FBP]_{eq} = \frac{[GAP]_{eq} \cdot [DHAP]_{eq}}{K_{eq}} = \frac{5 \cdot 10^{-5} M \cdot 5 \cdot 10^{-5} M}{6.7 \cdot 10^{-5} M} = 3.8 \cdot 10^{-5} M = 38 \,\mu$ M (see Cornish-Bowden, 1981, "Thermodynamic aspects of glycolysis")















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$$J_{\text{ATP}} \propto x \cdot (1 - x) \cdot \Delta G_{\text{driv}}$$



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<u>Conclusion</u>: Under physiological conditions in muscle cells, a ratio of 2 ATP per glucose is optimal!

Carbon balance in yeast

Data for yeast growing in fermentors, summary from many publications (from Rainer Machné, PhD thesis 2017)



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Approximately half is used as "work" (biomass production) and half released as heat!

$$\frac{dS}{dt} = \frac{d_{\text{int}}S}{dt} + \frac{d_{e}S}{dt} = \frac{\dot{S}_{\text{Prod}}}{T} + \frac{\dot{Q}}{T} + \sum_{i} s_{e,i} \dot{n}_{e,i} \leq 0$$

Internally produced entropy (>0) heat exchange metabolite exchange How do cells really export entropy? $G \approx 0$ $G \approx 0$ $G \approx 0$

Fig. 2. Coupling of catabolism and anabolism during growth.





The entropy balance of a living cell $\frac{dS}{dt} = \frac{d_{\text{int}}S}{dt} + \frac{d_eS}{dt} = \underline{\dot{S}}_{\text{Prod}} + \frac{\dot{Q}}{T} + \sum_i s_{e,i} \dot{n}_{e,i} \le 0$ Internally produced entropy (>0) heat exchange metabolite exchange **Biosynthetic reactions** How do cells really export entropy? (Anabolism) ∆**G** ≈ 0 aerobic growth (yeast) 700 ORGANISM Maximal ΔG , but no yield 600 -Δ_rH⁰_x, -Δ_rG⁰_x (kJ/C-mol) **Energy yielding reactions** ∆**G << 0** 500 (Catabolism) **Overall growth process:** (Metabolism) 400 -A,G°, Fig. 2. Coupling of catabolism and anabolism during growth. - 4, H⁰, 300 Yeast (aerobic) exports energy as heat 200 100 Maximal yield, but $\Delta G=0$: no growth 0.4 0.5 0.6 0.7 0.8 0.9 1.1 1 Y_{y/s} (C-mol/C-mol)

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Fig. 11. Schematic of enthalpic and entropic contributions to the driving force of microbial growth.

Summary

- Life is based on entropy export!
- Everything is entropy!
- Read Schrödinger's "What is Life?"

Merry Christmas!



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