





CEPLAS Cluster of Excellence on Plant Sciences

Mathematical Models of Plant Energy Metabolism

Towards synthetic starch

Oliver Ebenhöh



aSSB Strasbourg, 25.3.2015

Why do we need mathematical models?

- Simplified representation of reality
- Reduction to the essentials

"Simplicity is the ultimate sophistication" (Leonardo da Vinci)

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 \Rightarrow Models help to discover general principles!

Why do we need mathematical models?

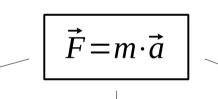
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- Reduction to the essentials

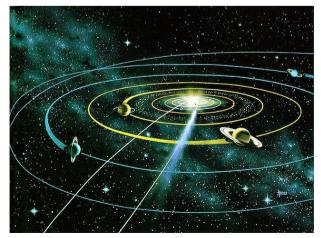
"Simplicity is the ultimate sophistication"

(Leonardo da Vinci)

Models help to discover general principles!

Example from physics:





www.thehungryandfoolish.com



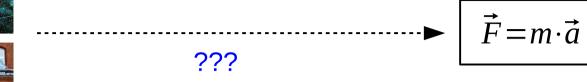


www.hh.schule.de

www.welt.de

How does one find principles (theory building)?









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$$\vec{F} = m \cdot \vec{a}$$

Intuition



Every model is a small step on this path

???



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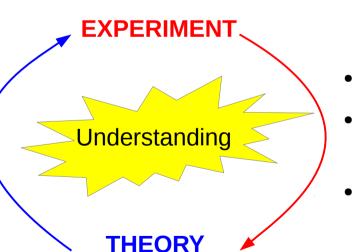
 $\vec{F} = m \cdot \vec{a}$



Every model is a small step on this path

???

- Model predictions / new hypotheses
- Suggestions for new experiments
- Improvement of experimental design



Initial model formulation

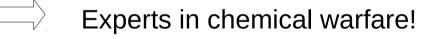
Intuition

- Confirmation / falsification of predictions
- New model assumptions

The Systems biology principle

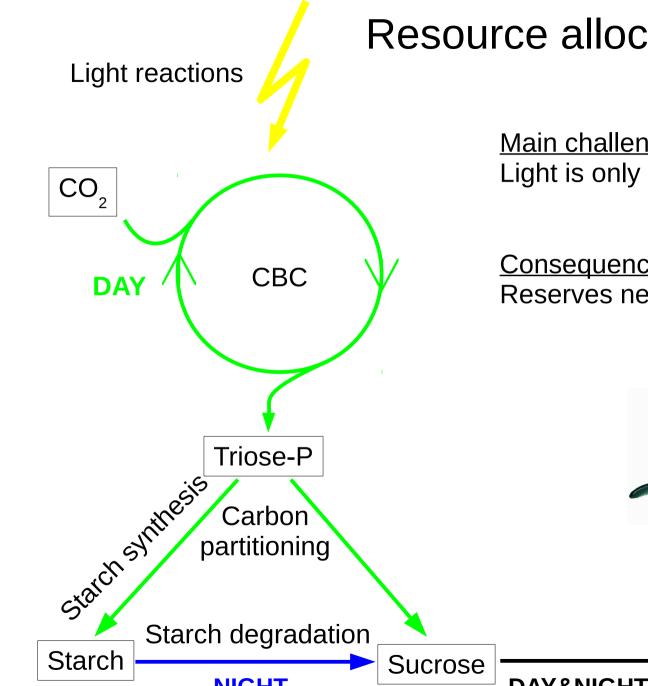
What's special about plants?

Photosynthesis
 Can't run away!



Estimated > 200,000 secondary metabolites!

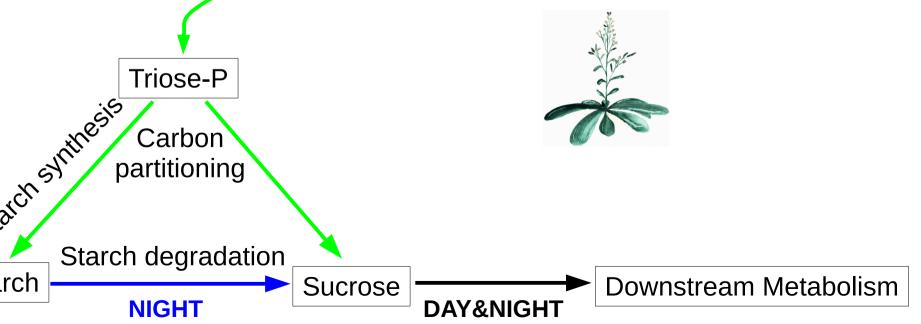




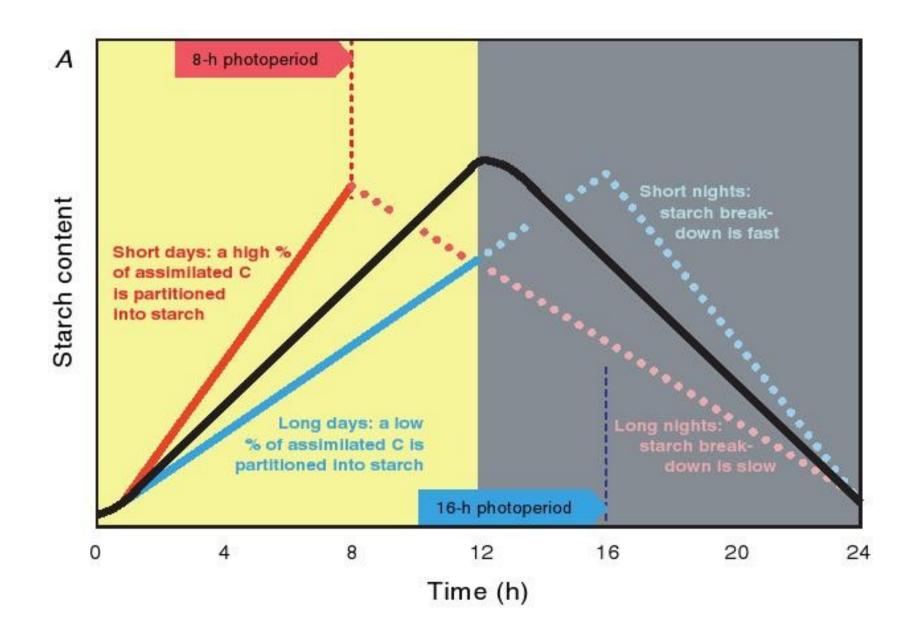
Resource allocation in plants

Main challenge: Light is only available during the day

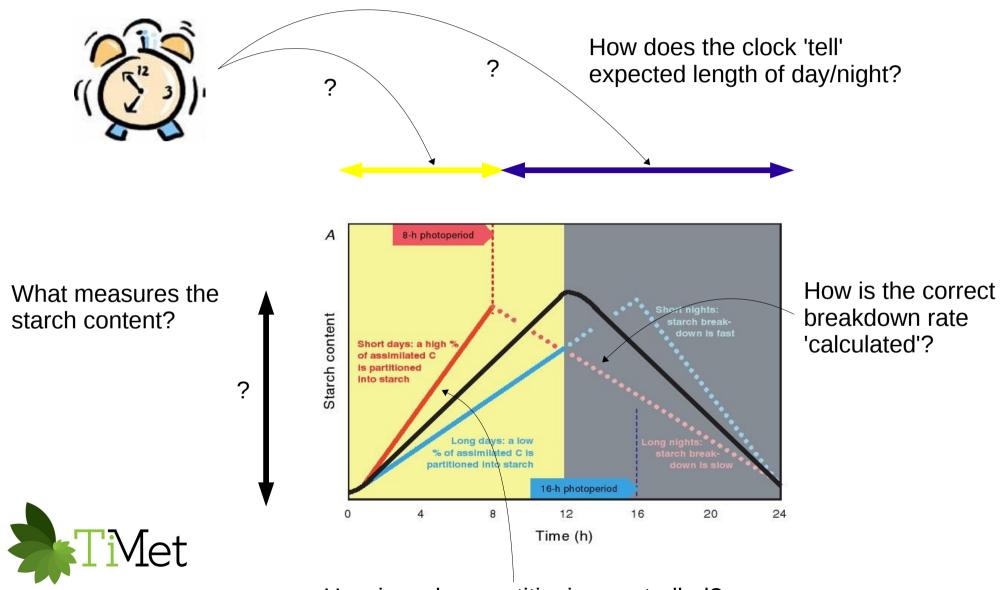
<u>Consequence</u>: Reserves need to be stored for the night



The diurnal turnover of starch

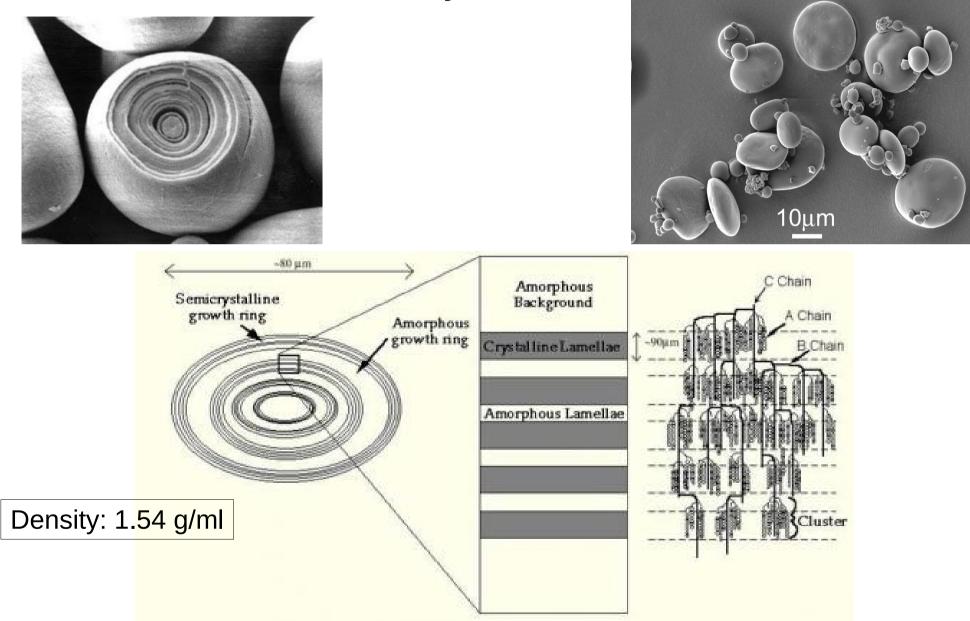


Open questions



How is carbon partitioning controlled?

Why starch?



The structure of starch allows for an extremely high energy storage density

Alternatives

energy content (kJ/g)

Carbohydrates	17
Lipids	38
Proteins	17
Alcohol	30

Possible advantages of starch

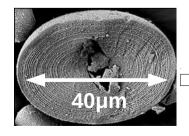
- low osmolarity
- large size
- high density

We (animals and fungi) predominantly use glycogen



big molecule (up to 10 MDa)

still small compared to starch



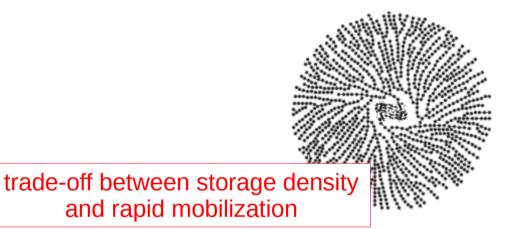
 $3 \cdot 10^{10} \text{ Da!!!}$

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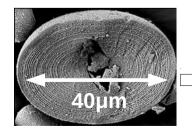


Possible advantages of starch

- low osmolarity
- large size
- high density

big molecule (up to 10 MDa)

still small compared to starch



 $3 \cdot 10^{10} \text{ Da}!!!$

optimised for storage density, slower deployment

How is starch made?

F6P

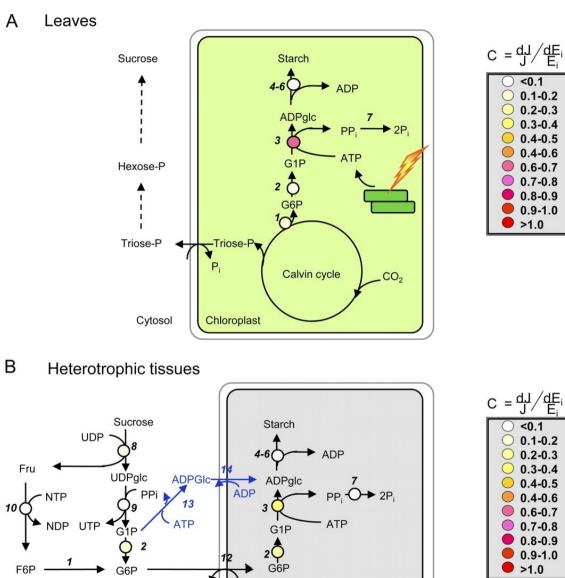
G6P

---- ATP

Cytosol

Respiration

(Mitochondria)



ADP

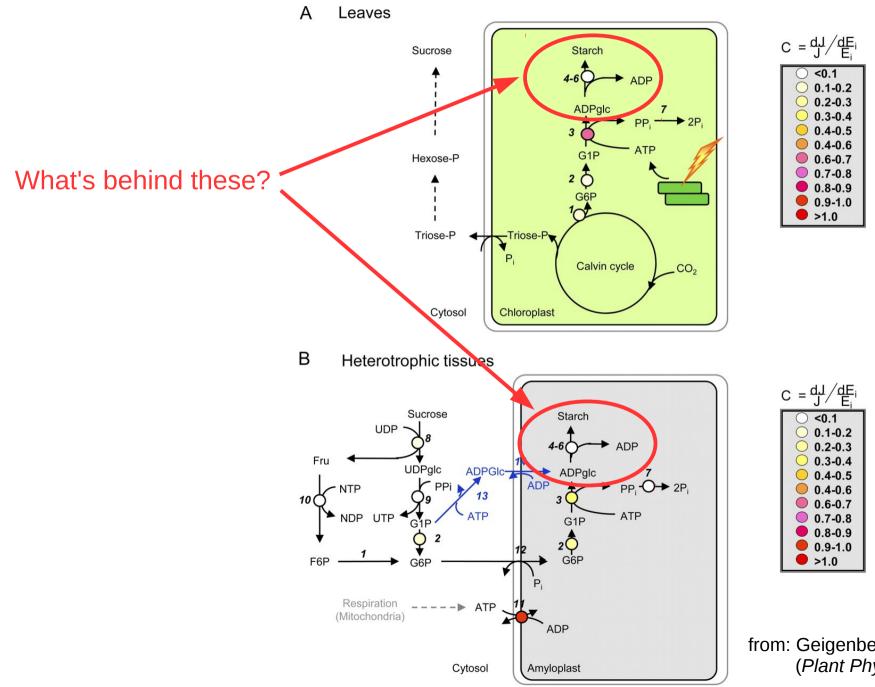
Amyloplast

from: Geigenberger 2011 (Plant Phys)

0.9-1.0

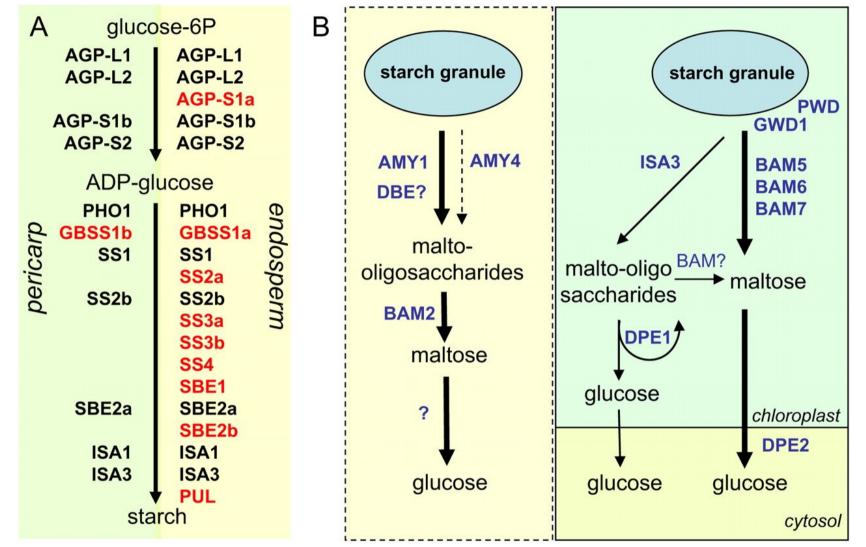
>1.0

How is starch made?



from: Geigenberger 2011 (Plant Phys)

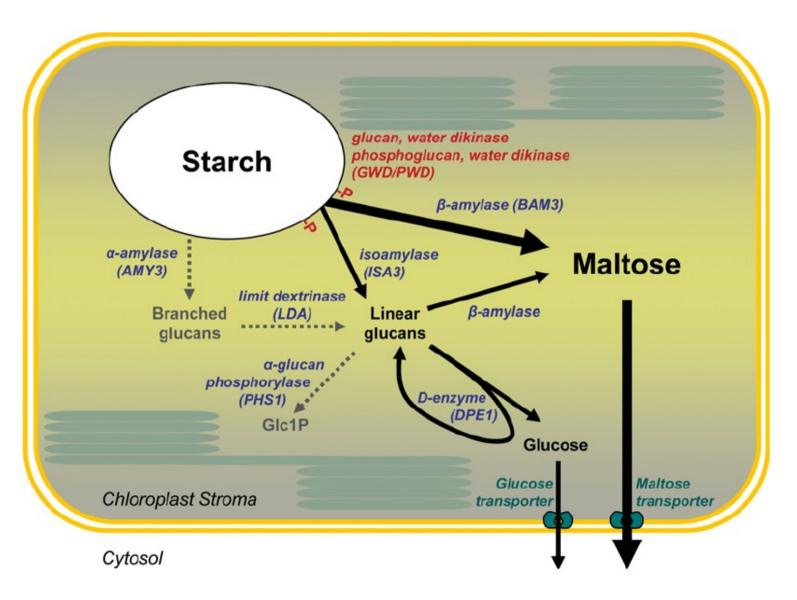
Many enzymes are involved in starch synthesis



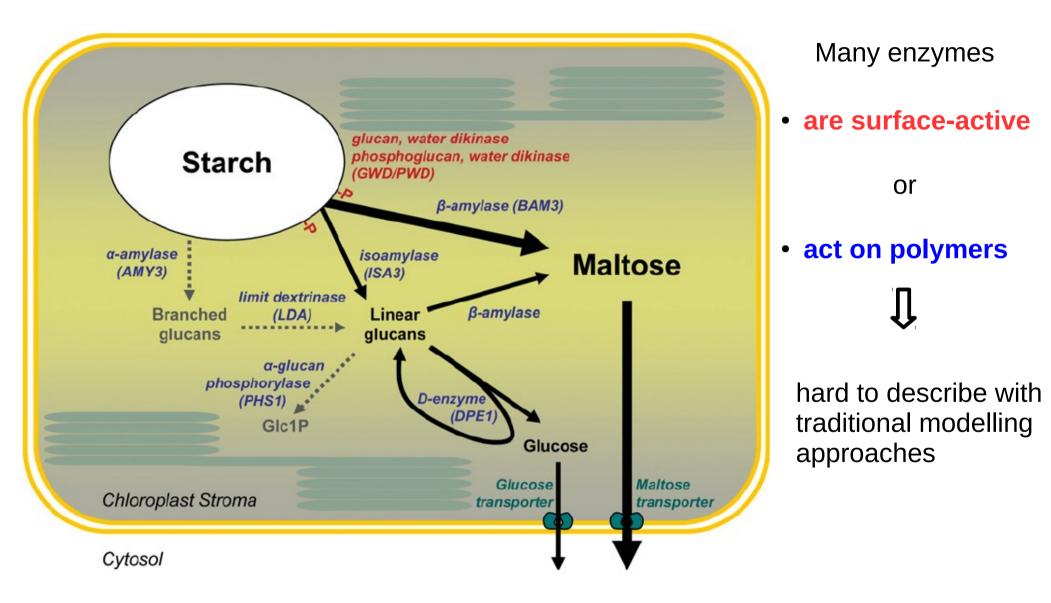
- starch synthases
- branching enzymes
- phosphorylases
- isoamylases

from: Radchuk et al 2009 (Plant Phys)

...and starch breakdown



...and starch breakdown

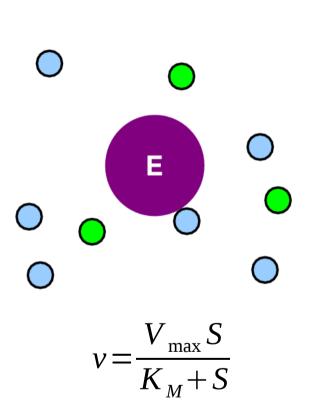


Challenges / Topics of lecture

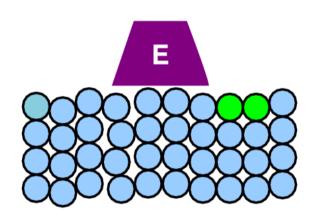
- 1. Surface-active enzymes
- 2. Polymer-active enzymes
- 3. Timing of starch metabolism

1. Surface-active enzymes

Rate laws for surfactive enzymes



dissolved substrate

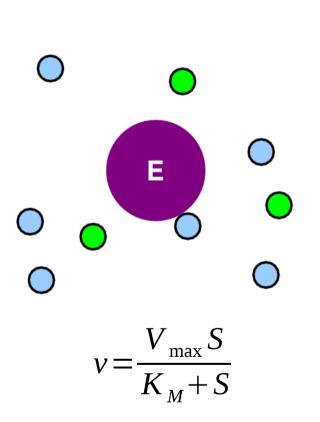


aggregated substrate

(with interfacial reaction space)

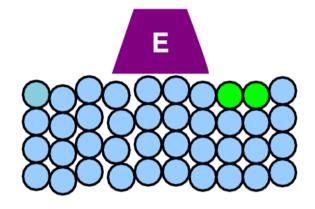
v = f(?)

Rate laws for surfactive enzymes



dissolved substrate

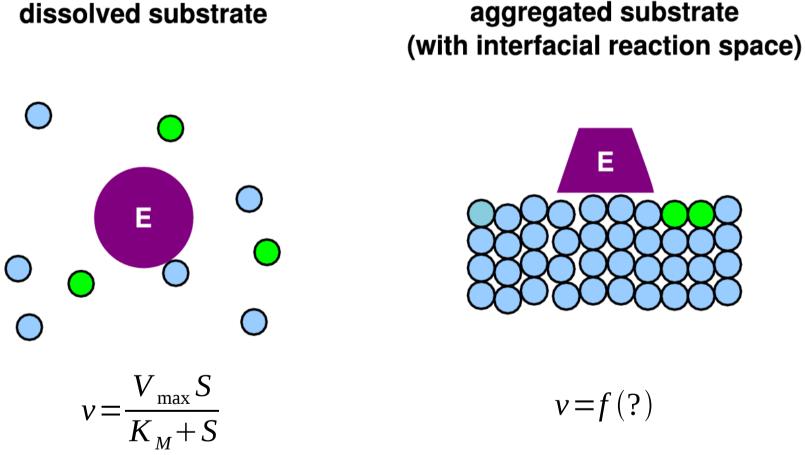
aggregated substrate (with interfacial reaction space)



v=f(?)

Reaction space confined to 2D!

Rate laws for surfactive enzymes

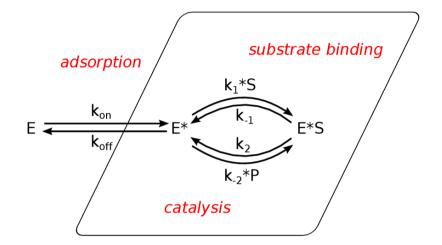


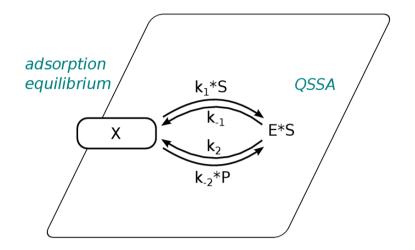
Reaction space confined to 2D!

Implications! - Fundamental differences to the classical case in solution:

- Relative activity dependent on enzyme concentration (jamming)
- Rate not independent on presence of other enzyme species! (competition)

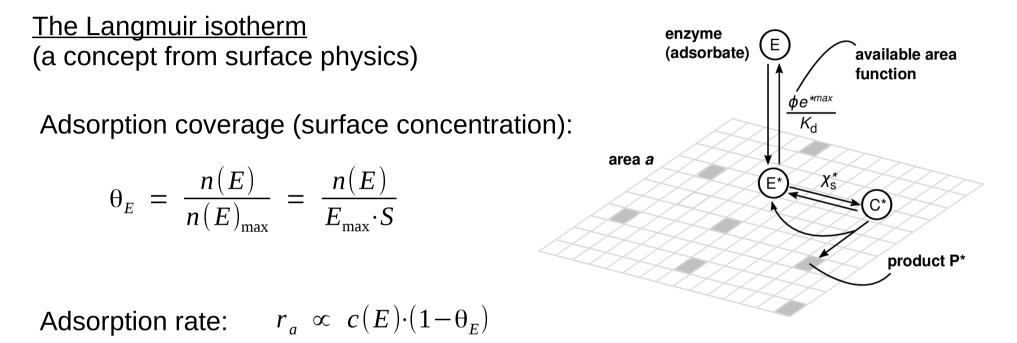
Derivation of a generic surfactive rate-law





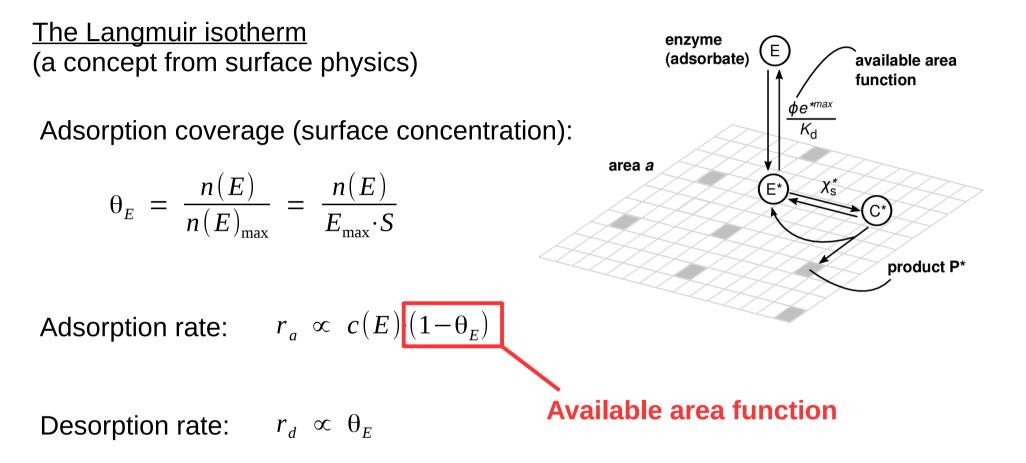
Kartal and Ebenhöh (2013) FEBS Letters – centenary issue commemorating Michaelis-Menten 'Kinetik der Invertinwirkung'

The adsorption equilibrium



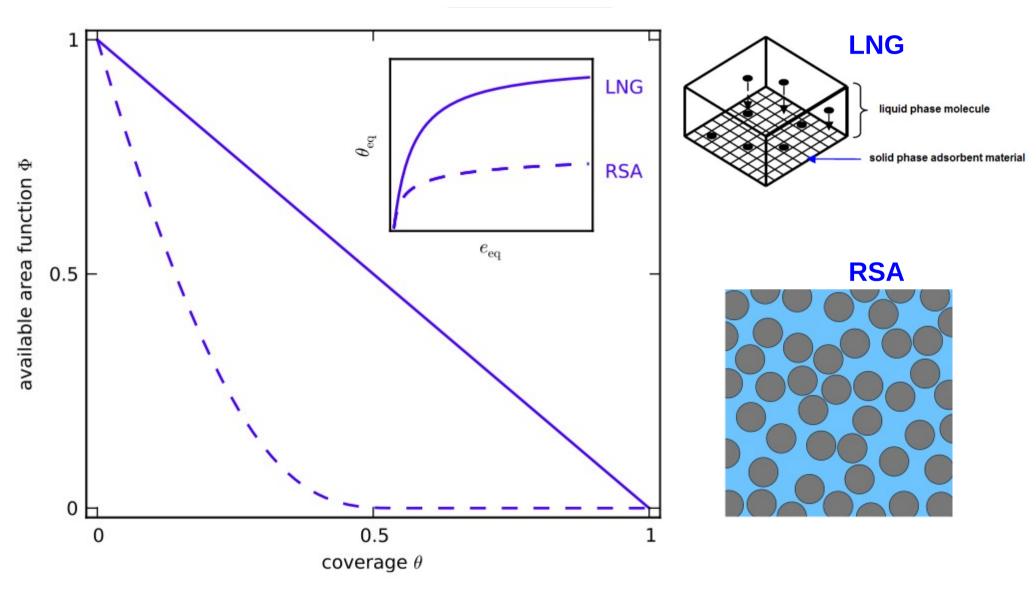
Desorption rate: $r_d \propto \theta_E$

The adsorption equilibrium

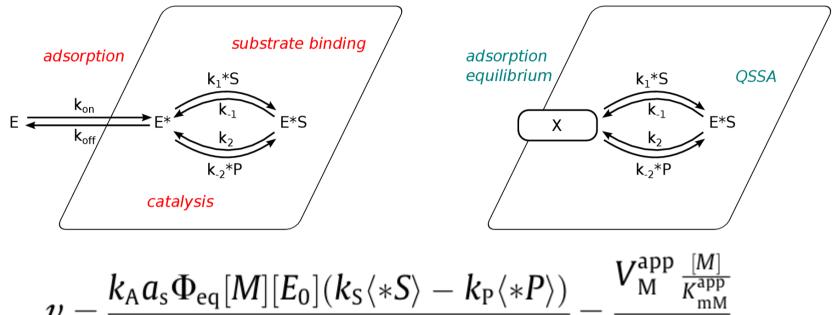


The adsorption equilibrium

Other adsorption models can give quite different results:



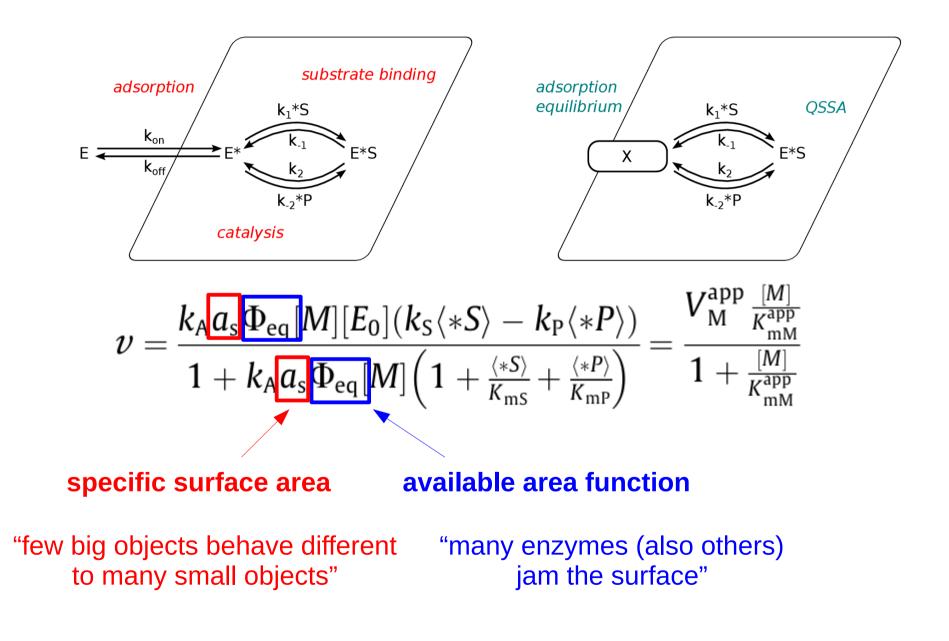
Derivation of a generic surfactive rate-law



$$\nu = \frac{1}{1 + k_{\rm A}a_{\rm s}\Phi_{\rm eq}[M]\left(1 + \frac{\langle *S\rangle}{K_{\rm mS}} + \frac{\langle *P\rangle}{K_{\rm mP}}\right)} = \frac{1}{1 + \frac{[M]}{K_{\rm mM}^{\rm app}}}$$

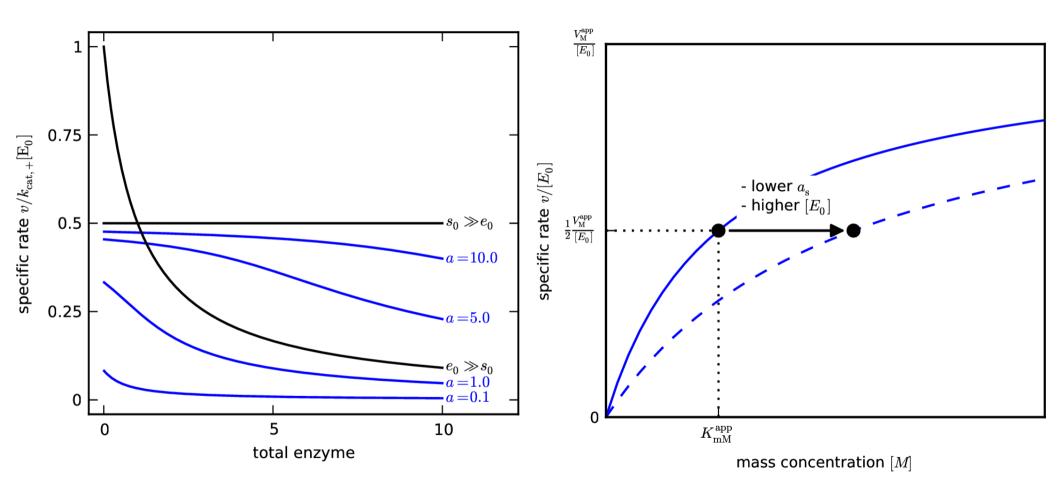
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Derivation of a generic surfactive rate-law



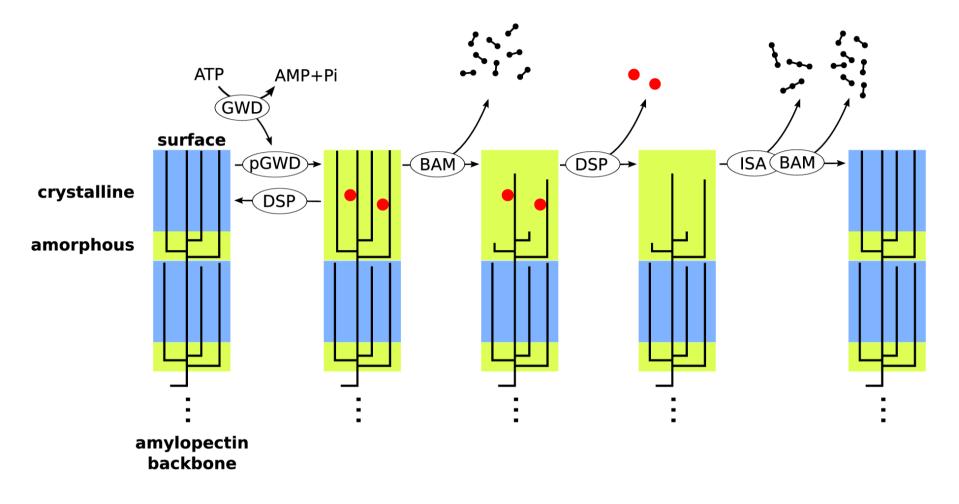
Kartal and Ebenhöh (2013) FEBS Letters – centenary issue commemorating Michaelis-Menten 'Kinetik der Invertinwirkung'

Consequences for experimental design



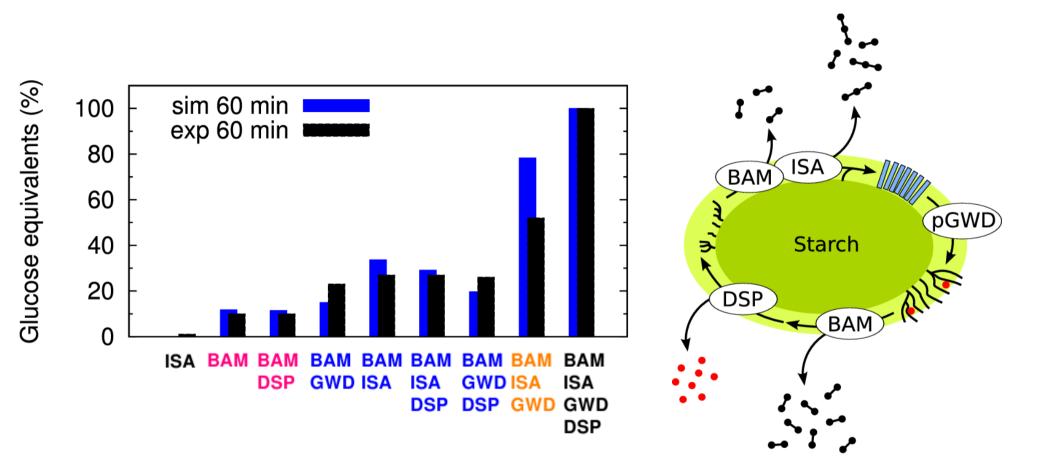
mass alone is insufficient!

A kinetic model of starch surface attack



- Disruption of crystalline surface by phosphorylation allows access for BAM and ISA
- Dephosphorylation by DSP enables further degradation

Simulations compared to experiment

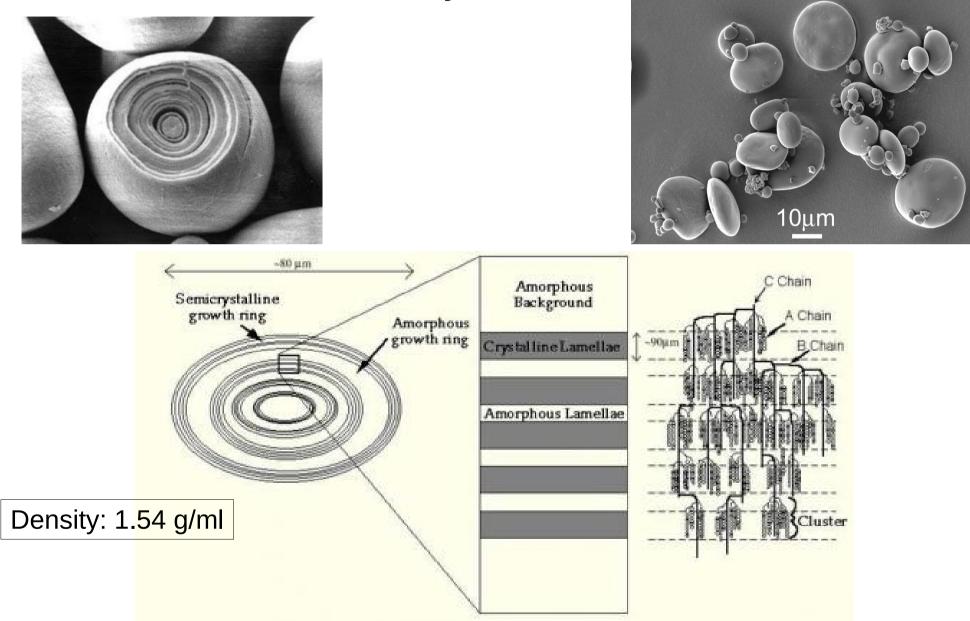


Good agreement with data from Kötting et al (2009) Plant Cell

But: only one time point!

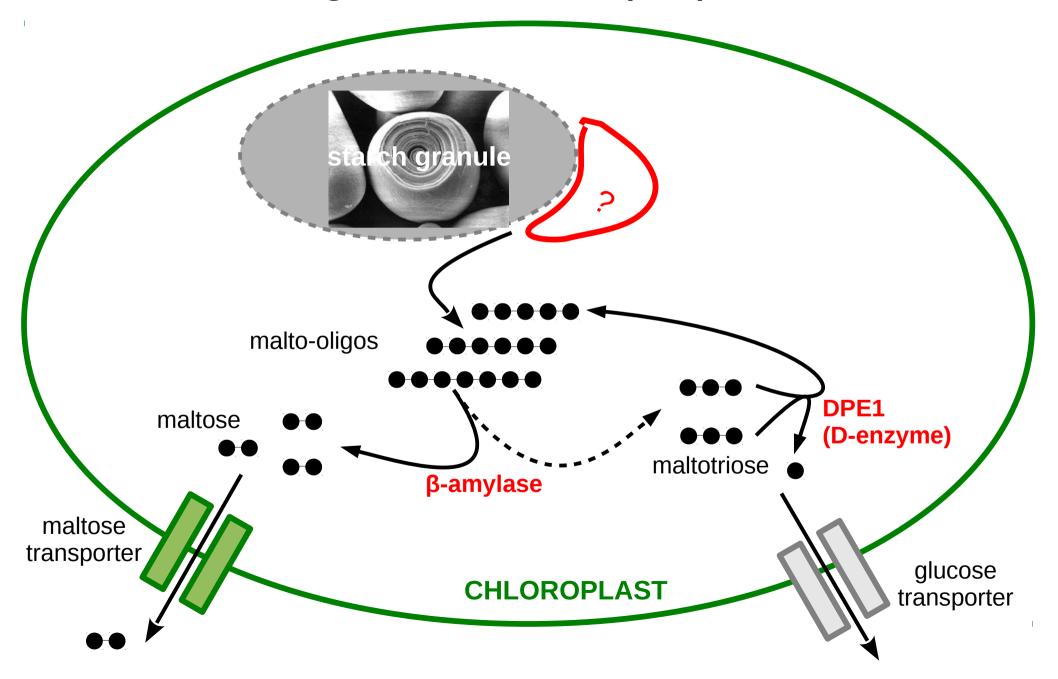
2. Polymer Biochemistry

Why starch?



The structure of starch allows for an extremely high energy storage density

Starch degradation - disproportionation



Disproportionating enzymes (D-enzymes)



EC: 2.4.1.25

but not only!

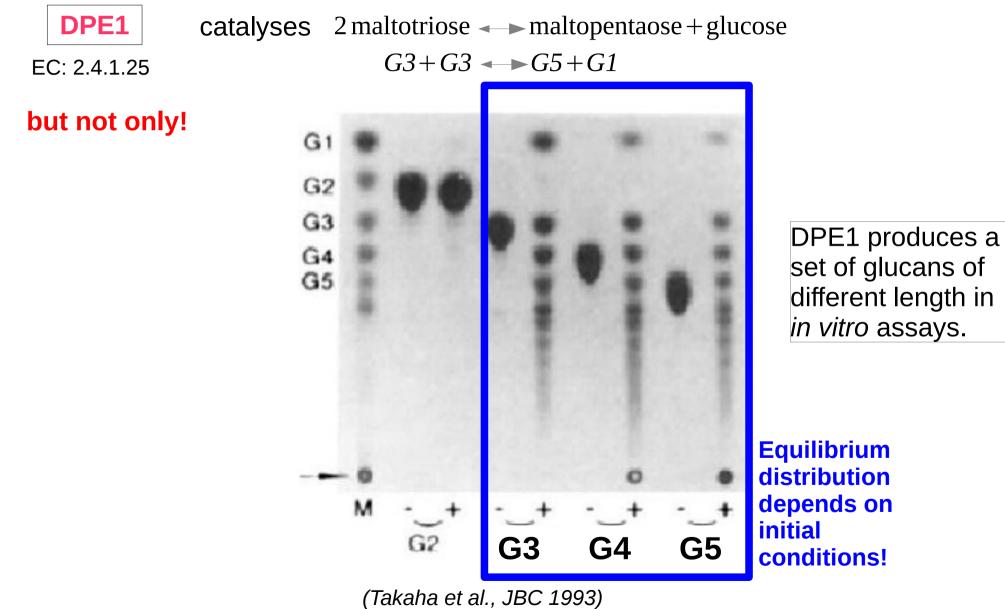
catalyses 2 maltotriose \checkmark maltopentaose + glucose $G3+G3 \checkmark G5+G1$

G1 G2 G3 Ğ4 **G**5 м G5 **G4** G3

DPE1 produces a set of glucans of different length in *in vitro* assays.

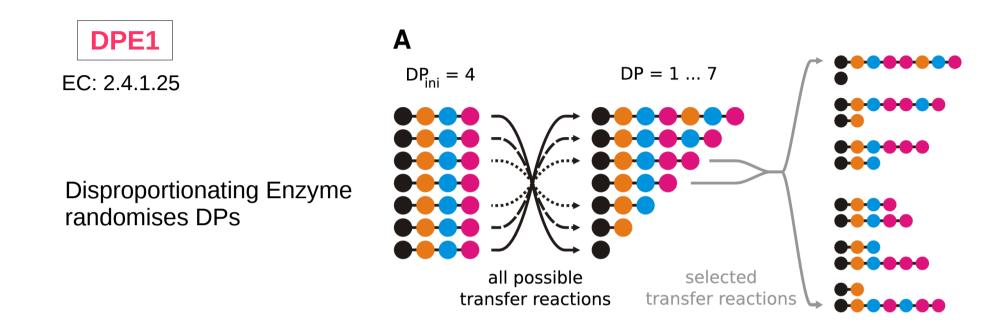
(Takaha et al., JBC 1993)

Disproportionating enzymes (D-enzymes)



*K*_{eq}???

Disproportionating enzymes (D-enzymes)



transfers glucosyl residues from one glucan to another: $G_n + G_m \longrightarrow G_{n-q} + G_{m+q}$

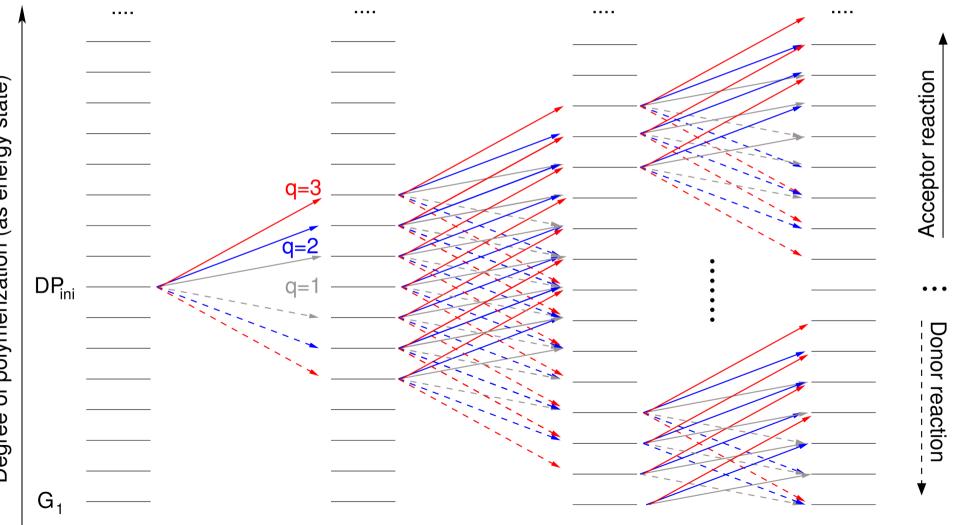
reaction must proceed towards a smaller Gibbs free energy : $\Delta G = \Delta H - T \Delta S < 0$

energy neutral (enthalpy of α -1,4-bond hydrolysis independent on position): $\Delta H = 0$ (Goldberg et al, 1992)

DPE1 maximises the entropy of the polydisperse reactant mixture

The thermodynamic picture

- Different DPs are interpreted as different energy states (energy of formation) •
- Enzymes mediate transitions between these states



Polydisperse mixtures as statistical ensembles

X_i : molar fraction of glucans with length *i* corresponds to occupation number of state *i*

The distribution $|X_i|$ fully characterises the polydisperse reactant mixture

The entropy of the statistical ensemble is $S = -\sum x_k \ln x_k$

Equilibrium is determined by maximal entropy:

$$S = -\sum x_k \ln x_k \rightarrow \max!$$

Maximum entropy principle under constraint that #bonds and #molecules is conserved!

conservation of #molecules:

conservation of #bonds:

$$\sum x_k = 1$$

$$\sum k \cdot x_k = b$$

determined by initially applied mixture of maltodextrins

Solution using Lagrangian multipliers: Necessary conditions are given by

$$\frac{\partial L}{\partial x_k} = 0 \quad \text{with} \quad L(x_k; \alpha, \beta) = \sum_k x_k \ln(x_k) + \alpha \left(\sum_k x_k - 1\right) + \beta \left(\sum_k k \cdot x_k - b\right)$$

 $\Leftrightarrow \ln(x_k) + 1 + \alpha + k \beta = 0$ for all k

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$$= \sum_{k} x_{k} = \frac{1}{Z} e^{-k\beta} \text{ with } Z = \sum_{k} e^{-k\beta}$$

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Analogy to statistical physics! There, $\beta = \frac{1}{k_B \cdot T}$

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Calculation of
$$\beta$$
: $-\frac{1}{Z}\frac{\partial Z}{\partial \beta} = b \iff \beta = \ln \frac{b+1}{b}$

Solution using Lagrangian multipliers: Necessary conditions are given by

$$\frac{\partial L}{\partial x_k} = 0 \quad \text{with} \quad L(x_k; \alpha, \beta) = \sum_k x_k \ln(x_k) + \alpha \left(\sum_k x_k - 1\right) + \beta \left(\sum_k k \cdot x_k - b\right)$$

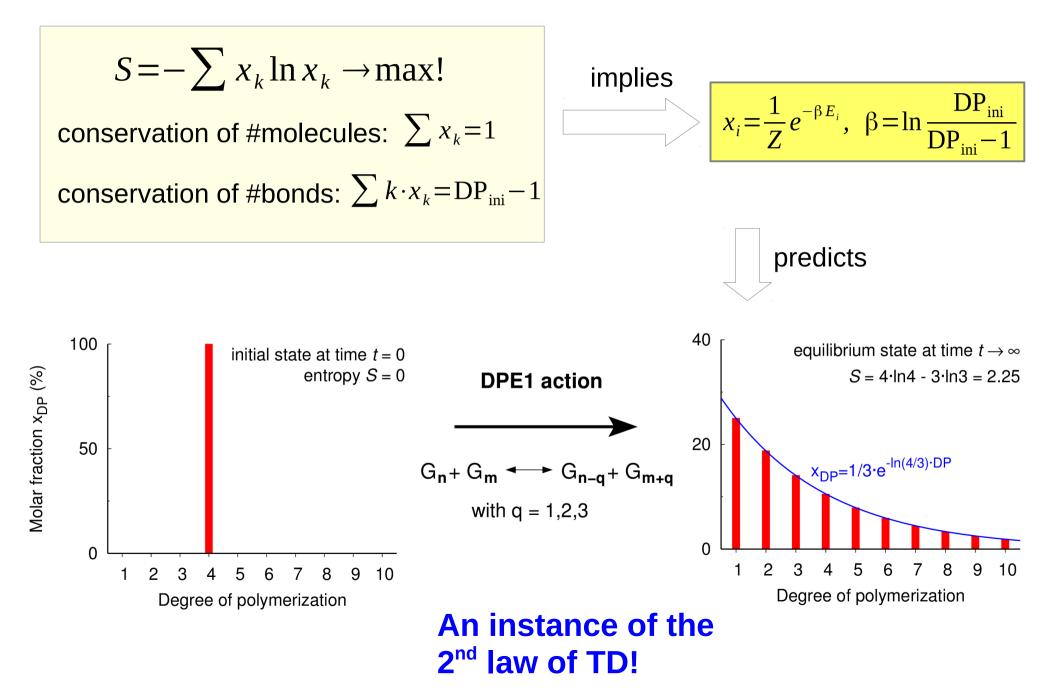
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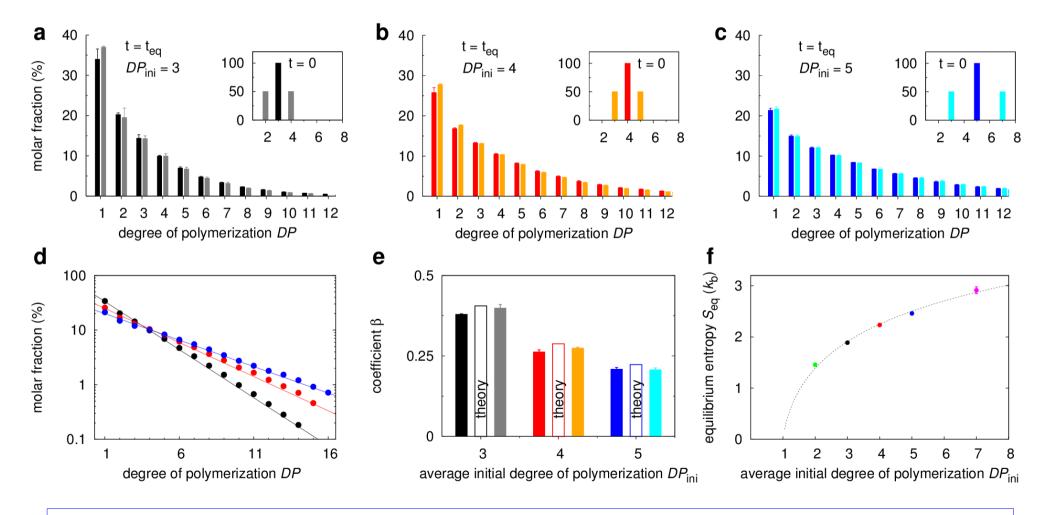
Maximal entropy in equilibrium: $S_{max} = (b+1)\ln(b+1) - b\ln b$



DPE1 is entropy driven

Experiments with Martin Steup, University of Potsdam

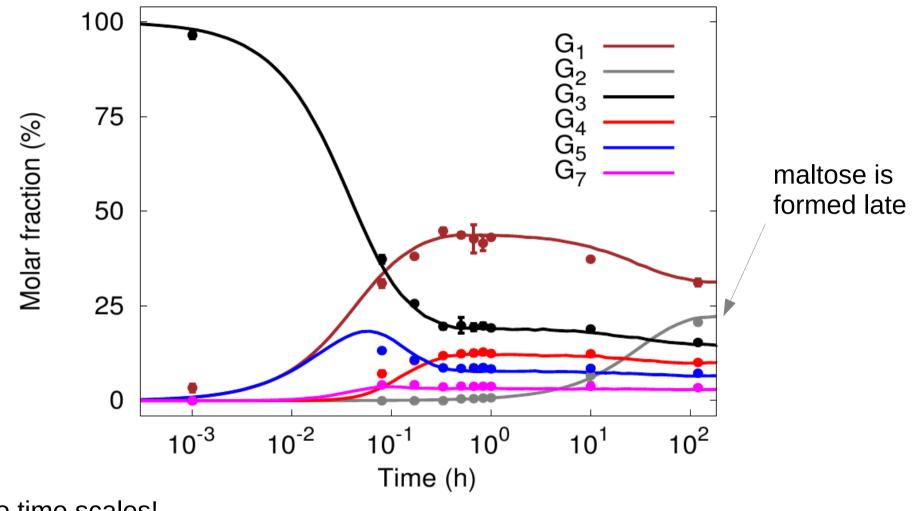
method: capillary electrophoresis



 β is a generalisation of the equilibrium constant for polydisperse mixtures

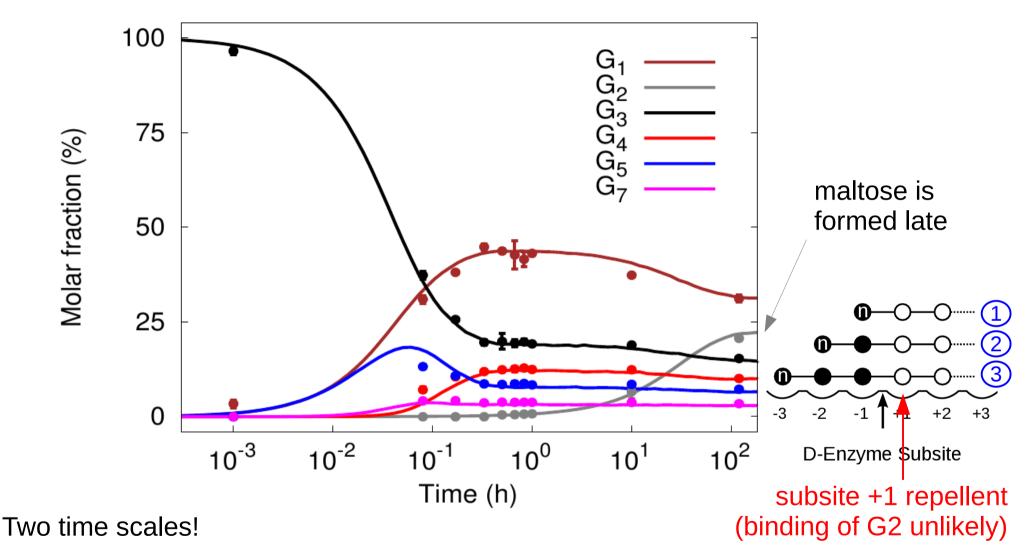
(Kartal et al, 2011, Mol Syst Biol)

The dynamics of DPE1



Two time scales!

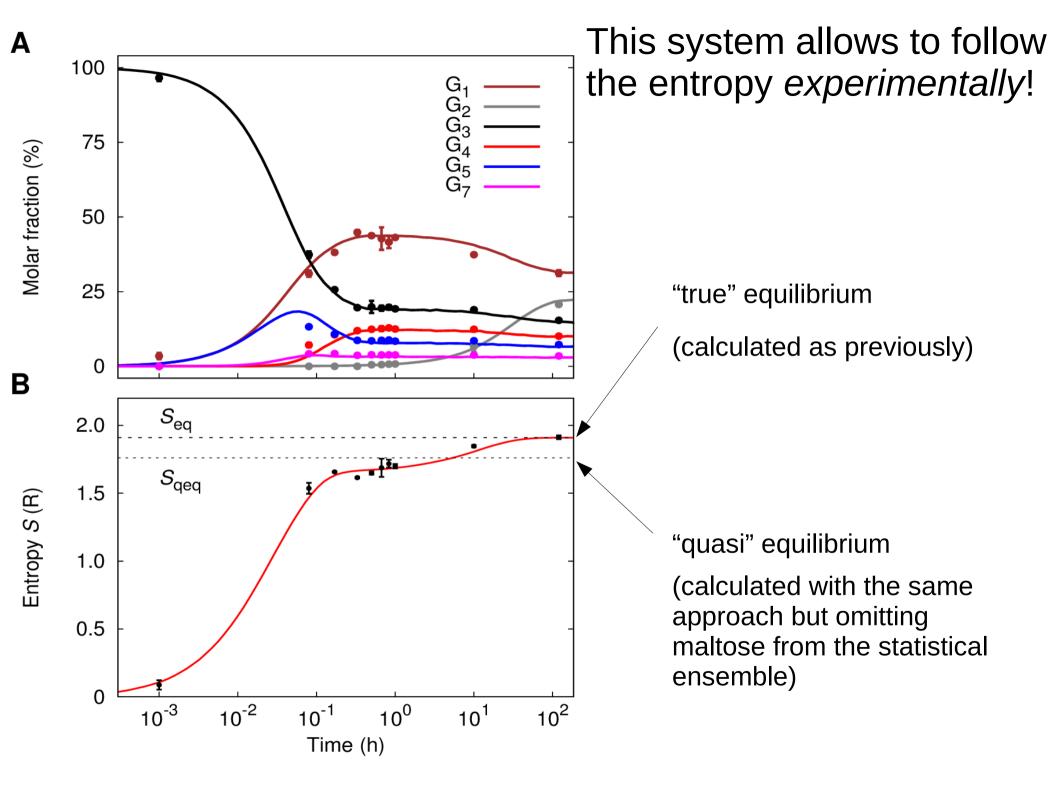
The dynamics of DPE1

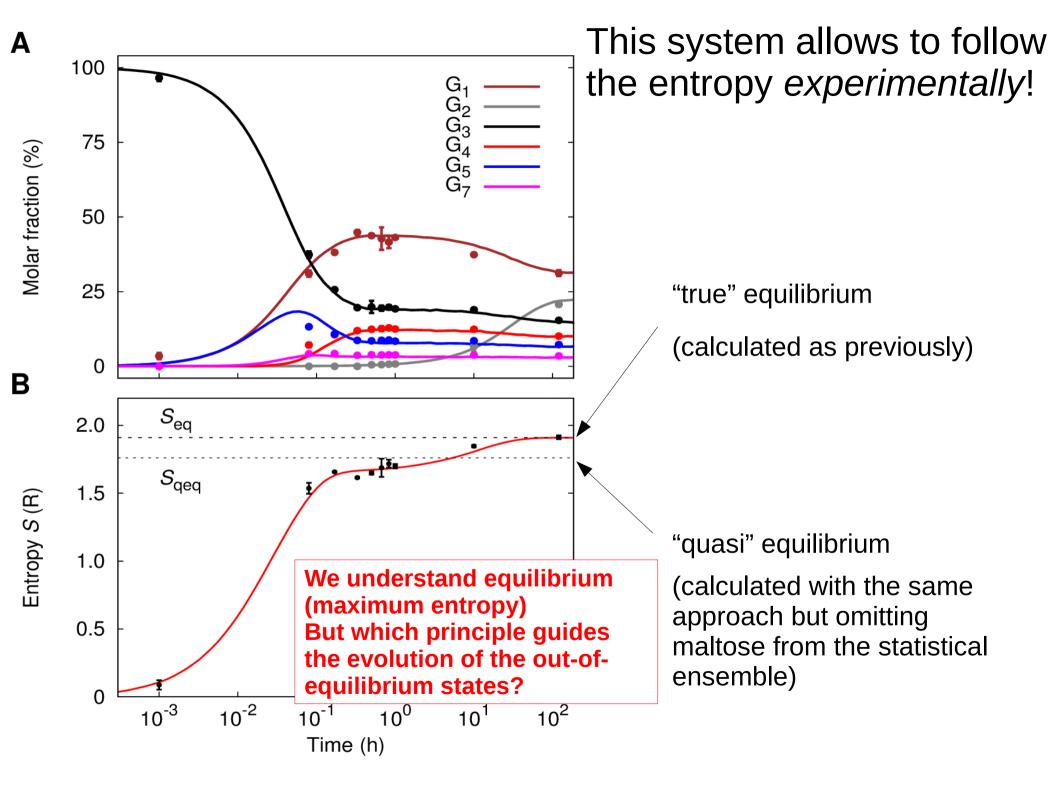


ratio 1:800

The simulations used 3 parameters:

- maximal turnover
- affinity for positional isomer 1
- affinities for positional isomers 2 and 3





Theory is also confirmed by DPE2

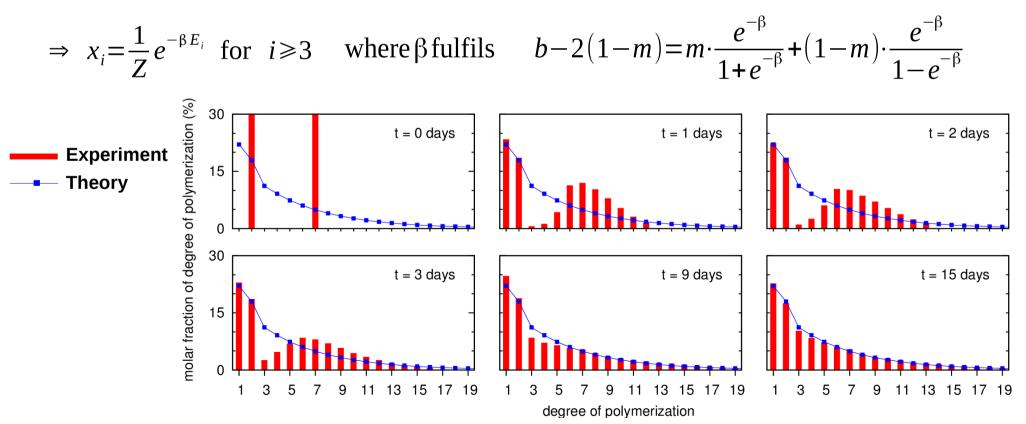
DPE2 vs DPE1

- transfers single glucosyl residues
- G2 only used as donor
- G3 only used as acceptor

<u>Generic reaction catalysed:</u>

 $G_n + G_1 \longleftarrow G_{n-1} + G_2$

Entropic principle: $S = -\sum_{k} x_{k} \ln x_{k} \rightarrow \max$ with one additional side constraint $x_{1} + x_{2} = m = \text{const.} \quad \left(\text{and} \sum x_{k} = 1; \sum k \cdot x_{k} = b \right)$



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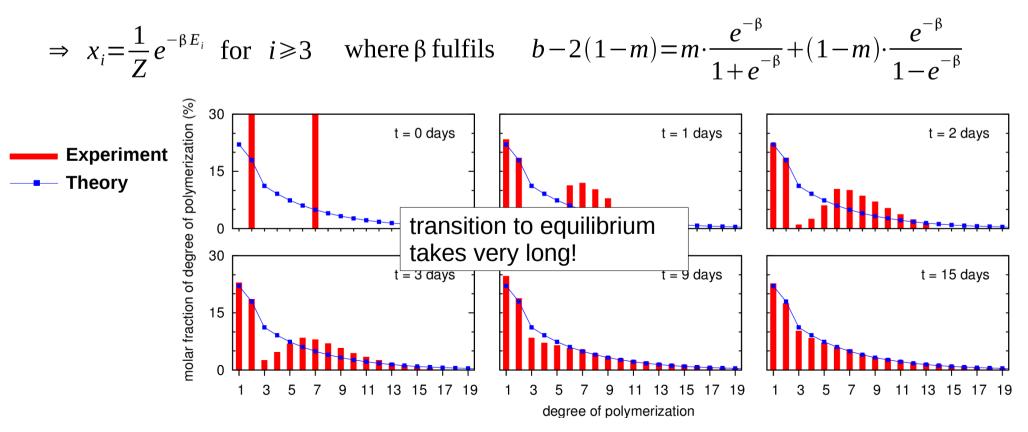
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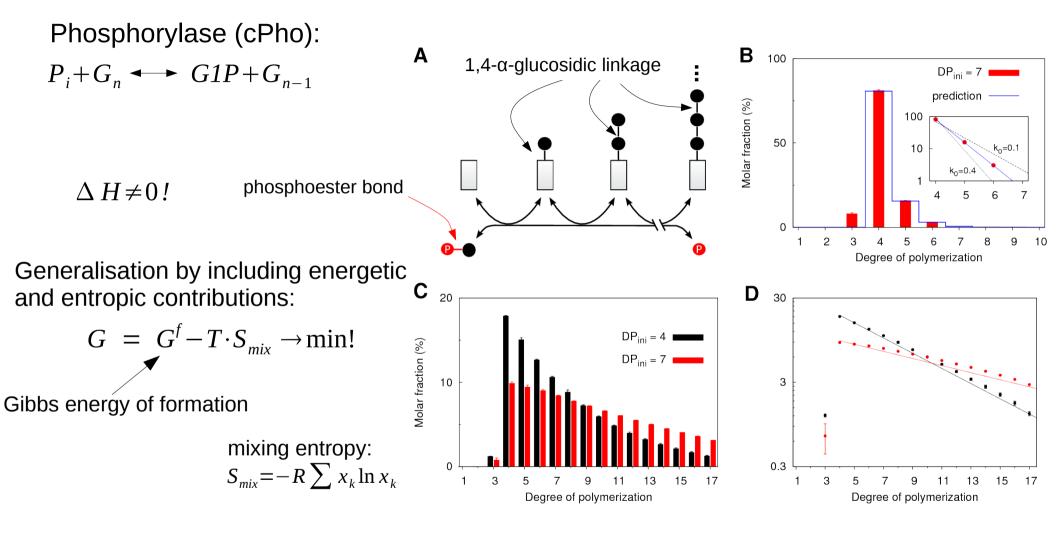
Entropic principle:

$$S = -\sum_{k} x_{k} \ln x_{k} \rightarrow \max$$
with one additional side constraint

$$x_{1} + x_{2} = m = \text{const.} \quad \left(\text{and} \sum x_{k} = 1; \sum k \cdot x_{k} = b \right)$$



Generalisation to non-zero enthalpy changes

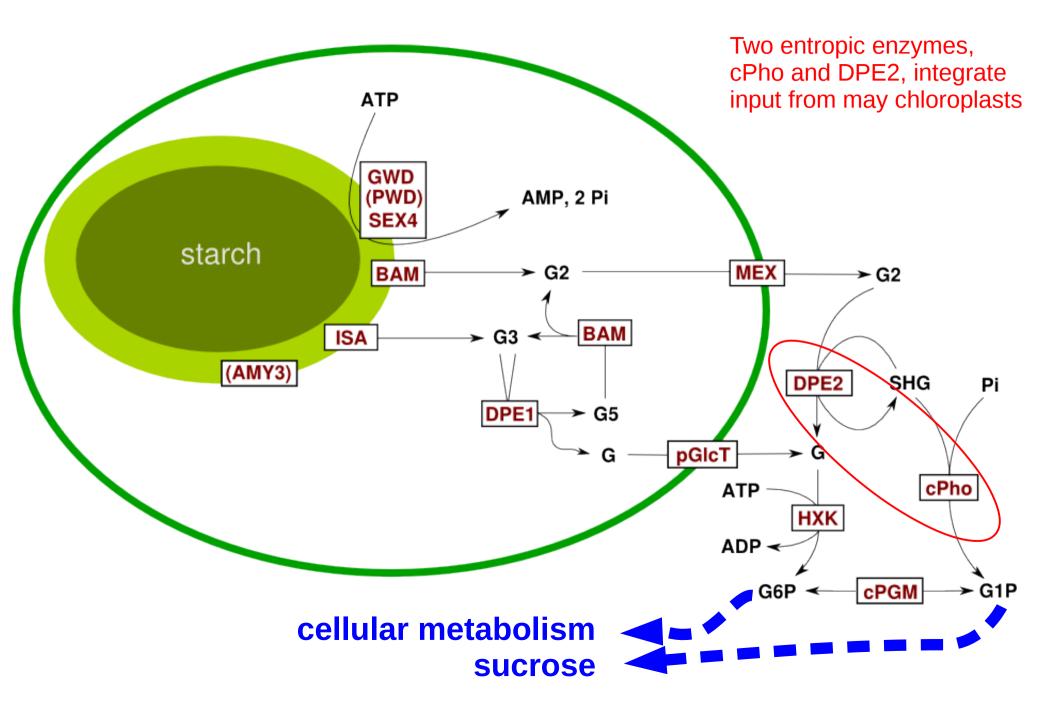


Prediction: Similar pattern as for DPE2

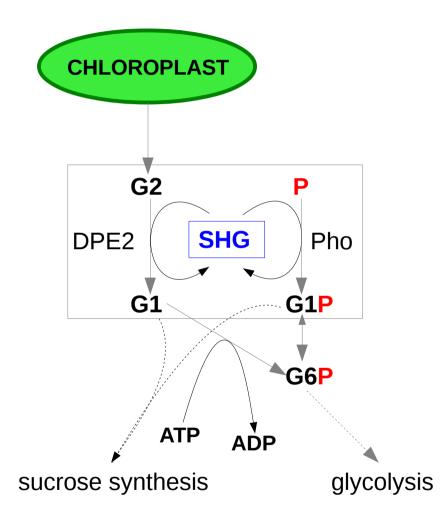
Experimentally confirmed.

(Kartal et al, Supp to MSB 2011; Ebenhöh et al, Proc 5th ESCEC 2013)

An entropy-driven buffer



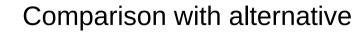
What is the role of the SHG pool?

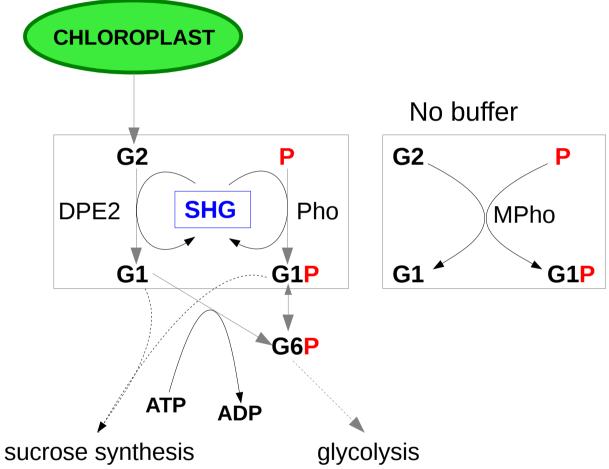


Two 'entropic' enzymes mediate the turnover of a polydisperse pool

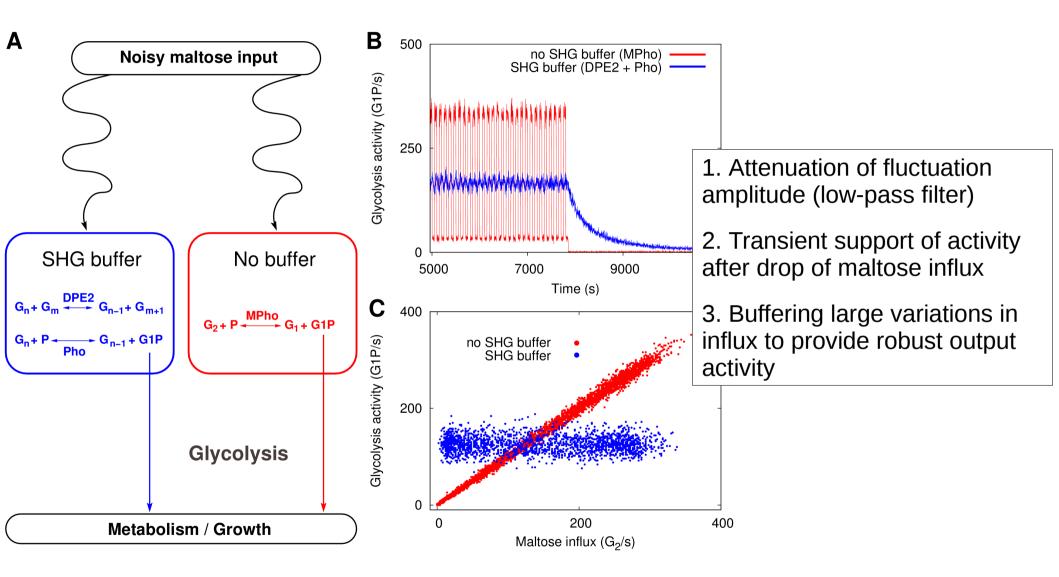
What is the advantage over other hypothetical systems?

What is the role of the SHG pool?

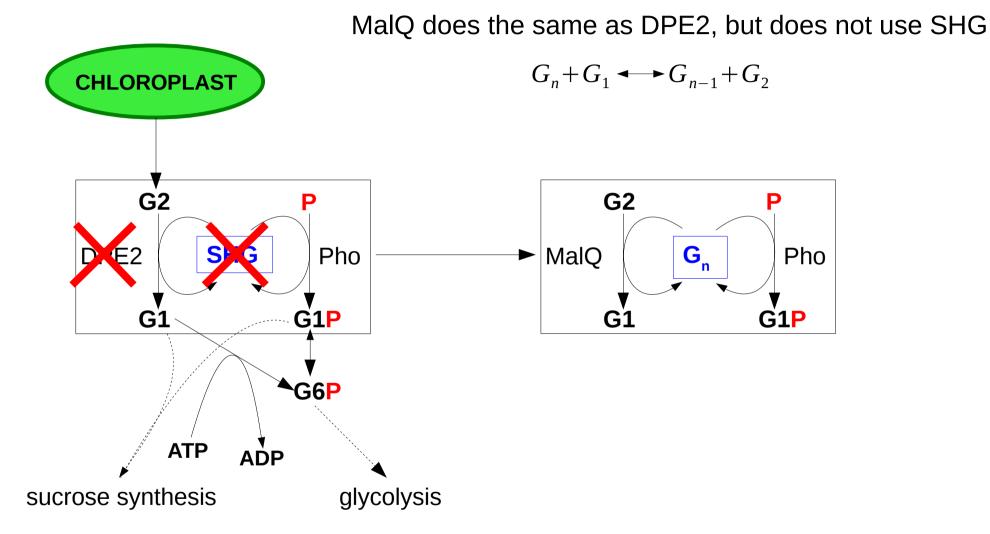




Polydisperse SHG pools increases robustness in vivo



Replacing DPE2 by MalQ



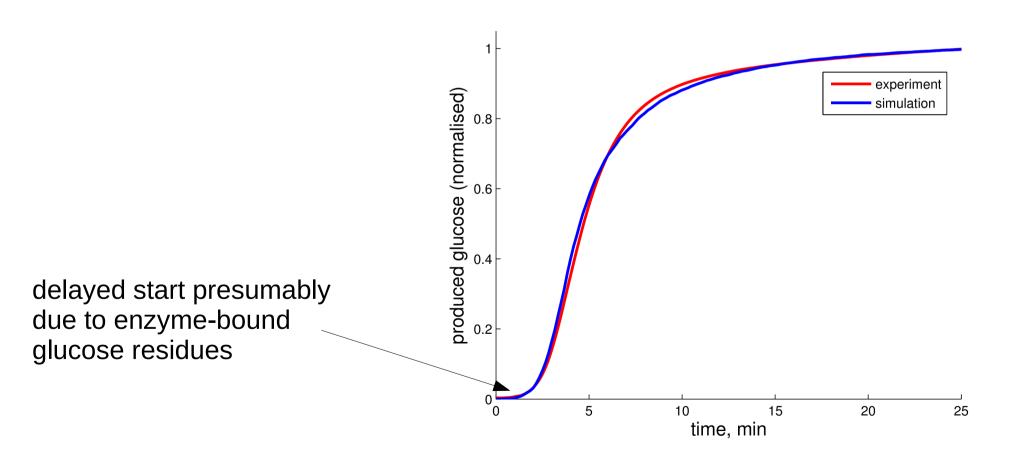
Simulating MalQ in vitro kinetics

In vitro system: DPE1 + HXK

 $G_1 \longrightarrow \emptyset$

 $G_n + G_1 \longleftarrow G_{n-1} + G_2 \quad n \neq 3$

Incubation with G₂ only!



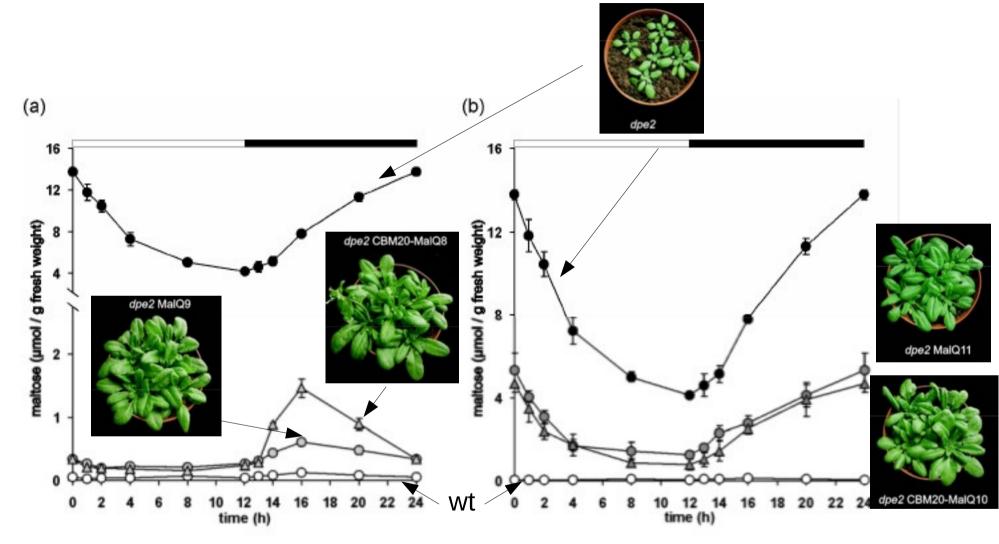
Moderate growth phenotype



(Julia Smirnova, PhD thesis; Ruzanski et al, JBC 2013)

complemented plants grow OK!

Maltose turnover

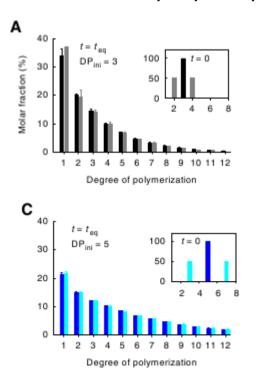


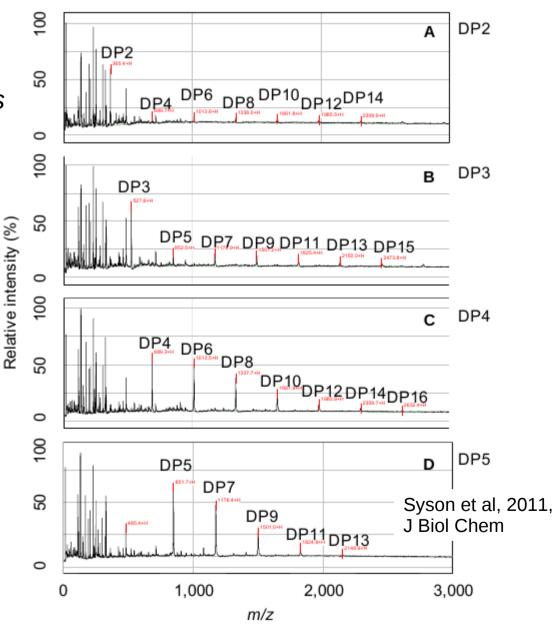
Where else do find entropic enzymes?

...for example

Maltosyltransferases in Streptomyces

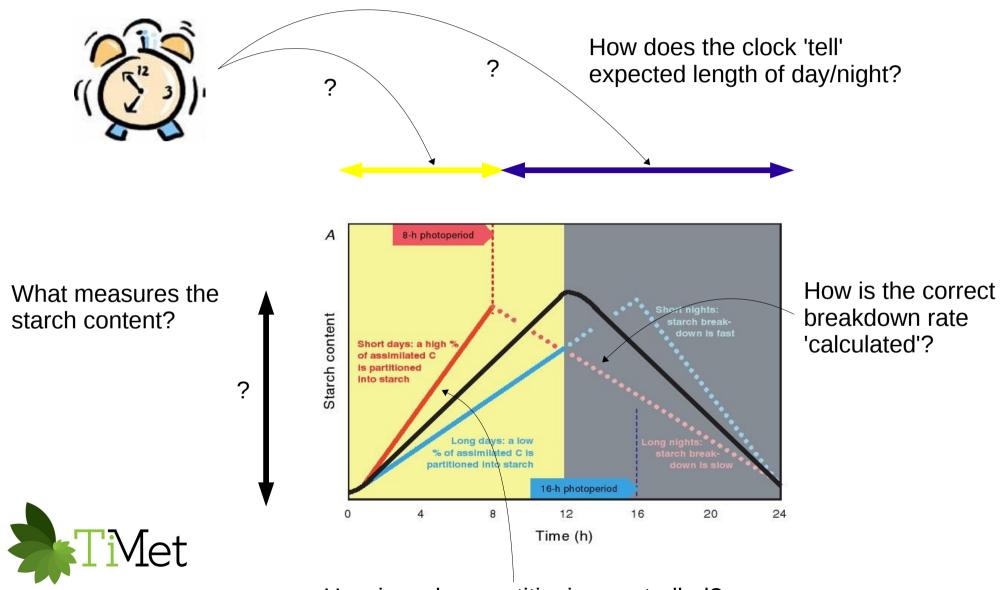
"Acceptor specificity" can be explained by entropic principles





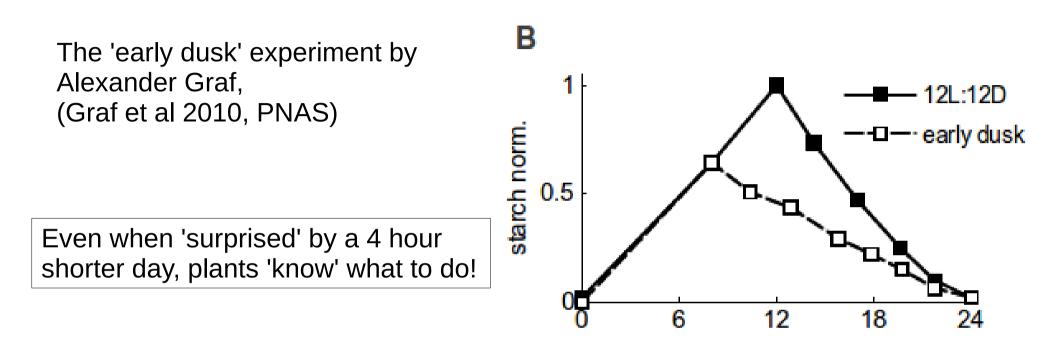
3. Timing of Metabolism

Open questions

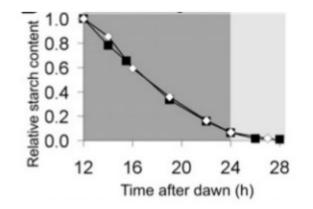


How is carbon partitioning controlled?

...even more mysteries...



The circadian clock is apparently important, because:



Plants cannot adapt to T-cycles different than 24h!

Building a mathematical model

Known:

- Metabolism
- Circadian clock



Alexandra Pokhilko

Unknown:

- Regulation of starch synthesis
- Regulation of starch breakdown
- How is starch content measured?

Challenges:

1. The model must combine known systems with plausible, but hypothesised regulatory mechanisms

2. To keep the model tractable, we need to find a compromise between detailedness and simplification

Seaton et al, 2013, *Roy Soc Interface*; Pokhilko et al, 2014, *Mol BioSyst*; Pokhilko et al, 2015, *Roy Soc Interface*

How to regulate starch degradation? B $1 \rightarrow 12L:12D$ $0.5 \rightarrow 0.5 \rightarrow 0.5$

Simplest solution:

Auxiliary compound X (e.g. active form of starch degrading enzyme):

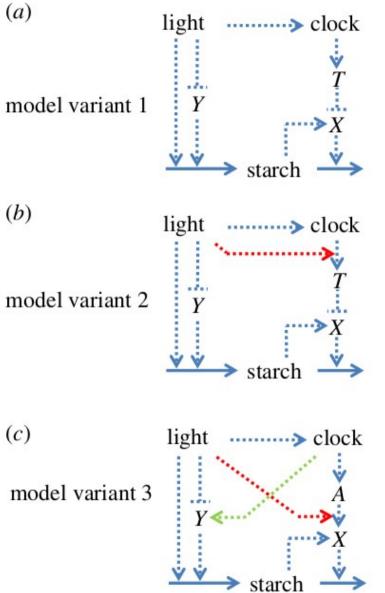
$$\frac{dX}{dt} = k_1 S - k_2 X T$$

Rapid activation/deactivation:

$$\frac{dX}{dt} = 0 \iff X = \frac{k_1}{k_2} \cdot \frac{S}{T}$$

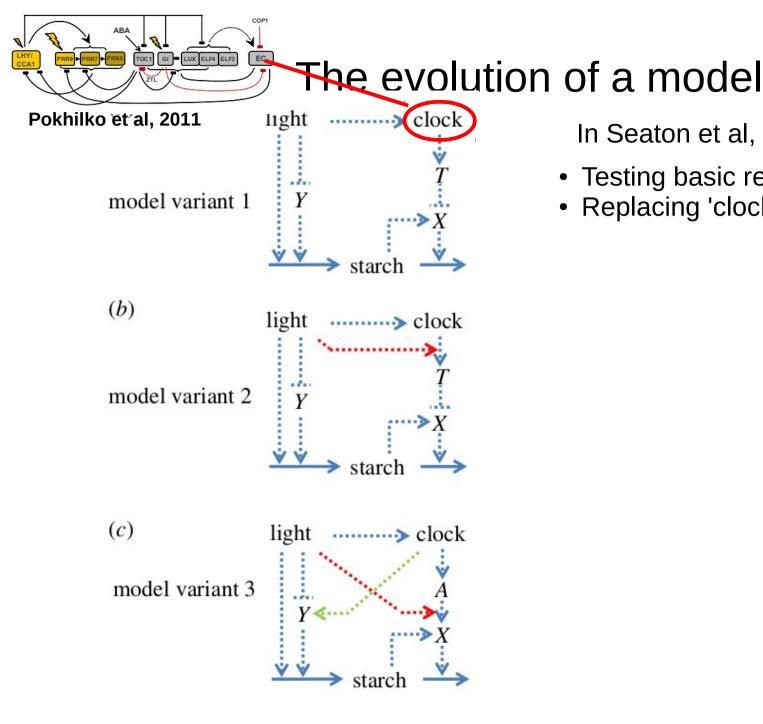
Scaldione et al (2013), eLife: Arabidopsis plants perform arithmetic division to prevent starvation at night

The evolution of a model



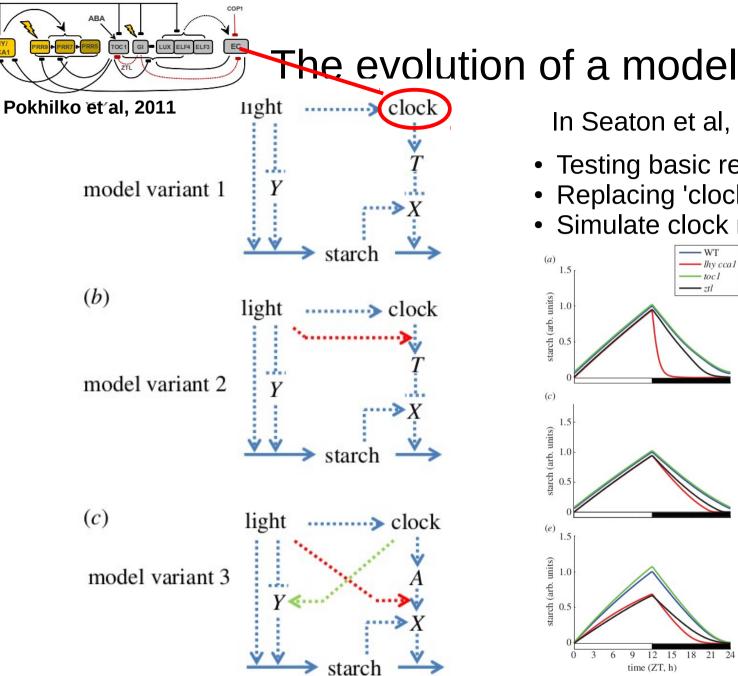
In Seaton et al, 2013:

• Testing basic regulatory mechanisms



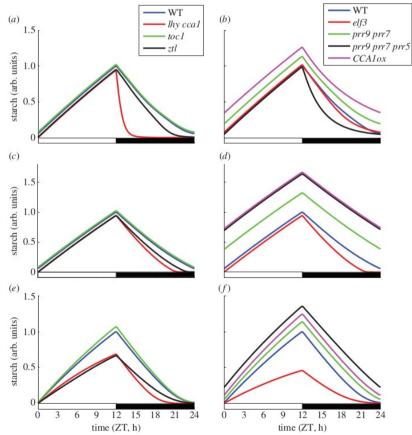
In Seaton et al, 2013:

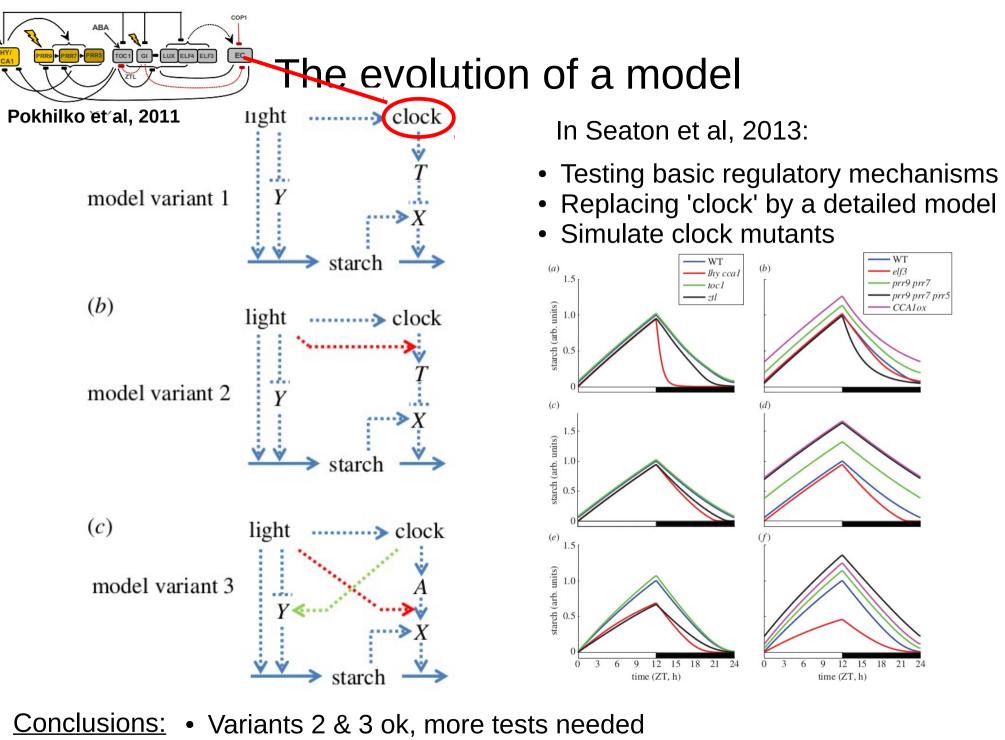
- Testing basic regulatory mechanisms
- Replacing 'clock' by a detailed model



In Seaton et al, 2013:

- Testing basic regulatory mechanisms
- Replacing 'clock' by a detailed model
- Simulate clock mutants





• Components A,X,Y remain hypothetical

cytosol chloroplast respir TP (TP Pi co, **F16P** G CBC **cFBPase** G6P TP G1P М F16P F6PK F6P (UGPG) F26P sFBPase SPS F26PP F6P G suc м G6P night bam day export G1P starch both

Adding more details of metabolism

- Carbon fixation
- Starch synthesis
- Starch breakdown
- Sucrose synthesis
- Sucrose export

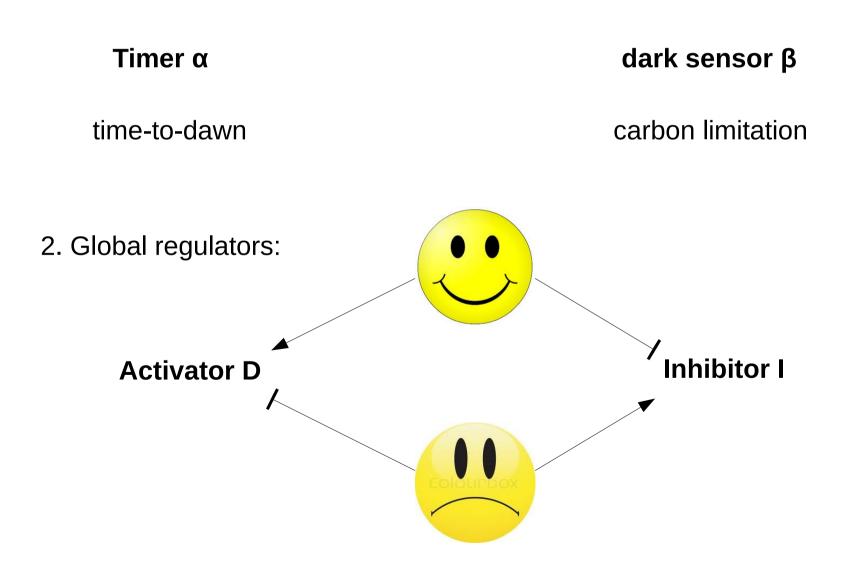
Pokhilko et al, 2014, Mol Biosystems

AGPase

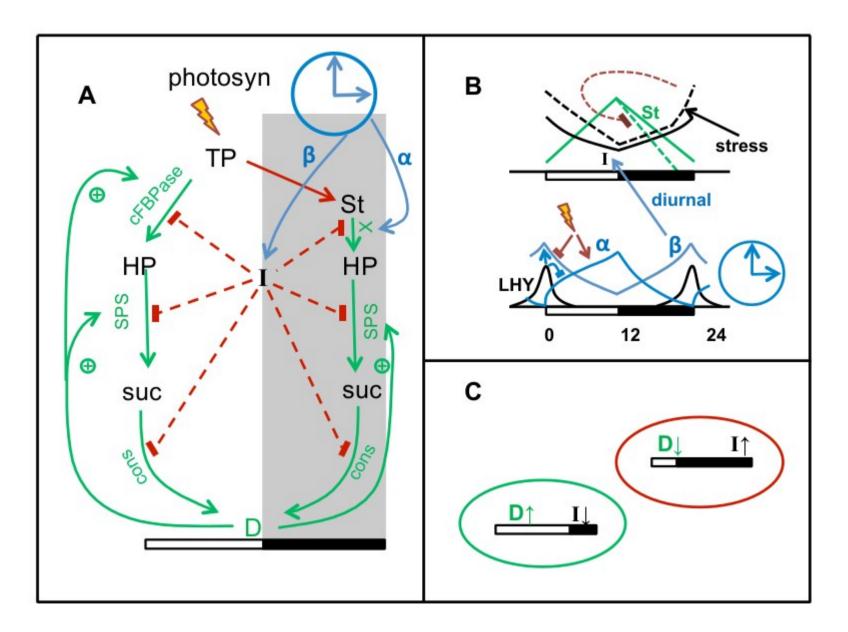
Include key steps but simplify pathways!

Model assumptions (postulates)

1. Key sensors:

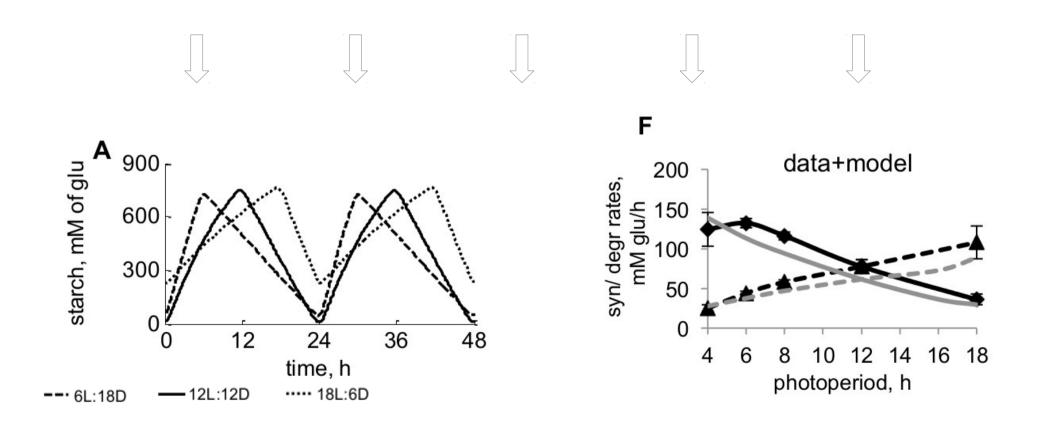


Regulatory principles



Simulations wild-type

Regulatory principles allow to explain wild-type starch turnover under various photoperiods



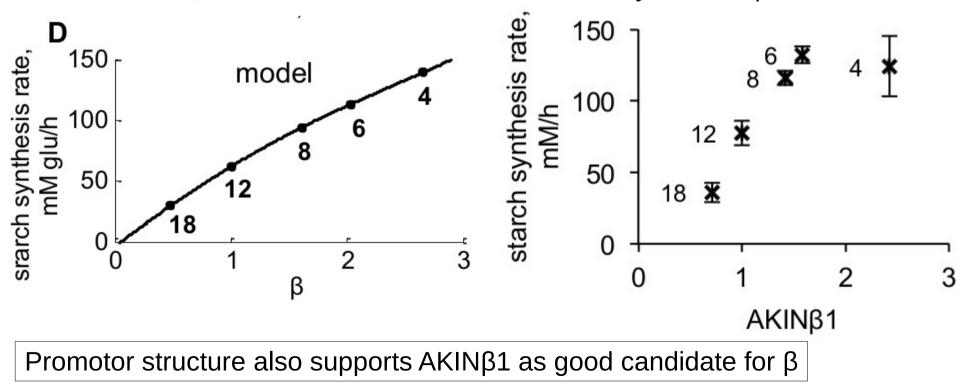
What are the unknown components?

Model allows to make predictions of their behaviour

> Helps to identify candidates from expression / proteomics data

For example, the component β :

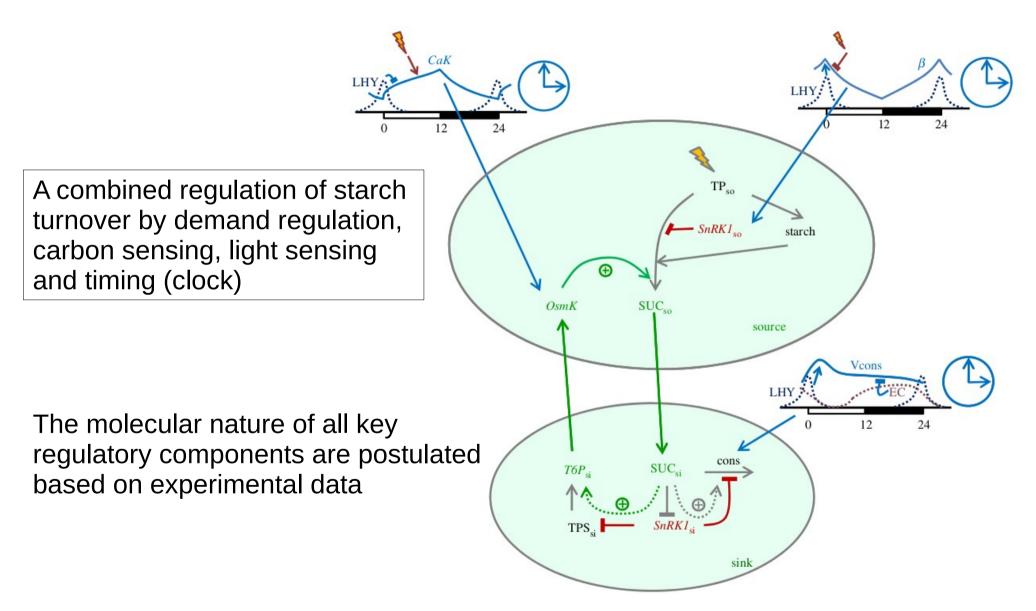
Predicted peak-levels at dawn



Microarray data for β -subunit of SNRK1

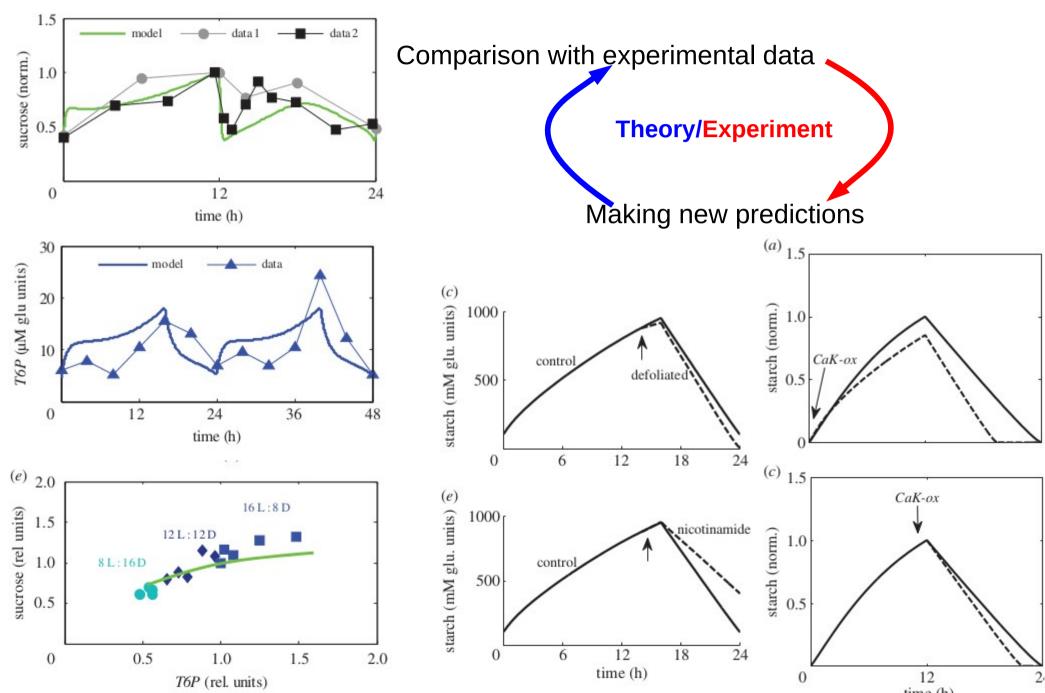
Other regulatory components still unknown!

The third generation



Pokhilko et al, 2015, Roy Soc Interface

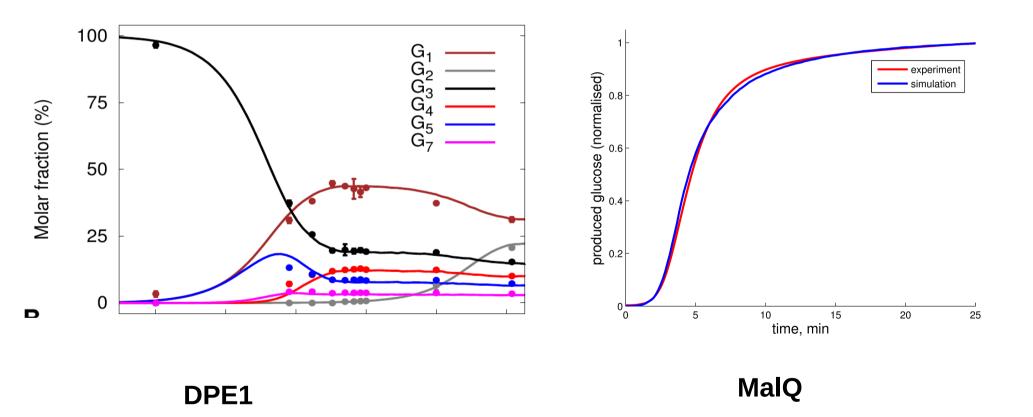
Improved results and new predictions



Outlook – towards designing starch

1. Understand and describe polymer-active enzymes

1. Understand and describe polymer-active enzymes



1. Understand and describe polymer-active enzymes

OK

Require more data:

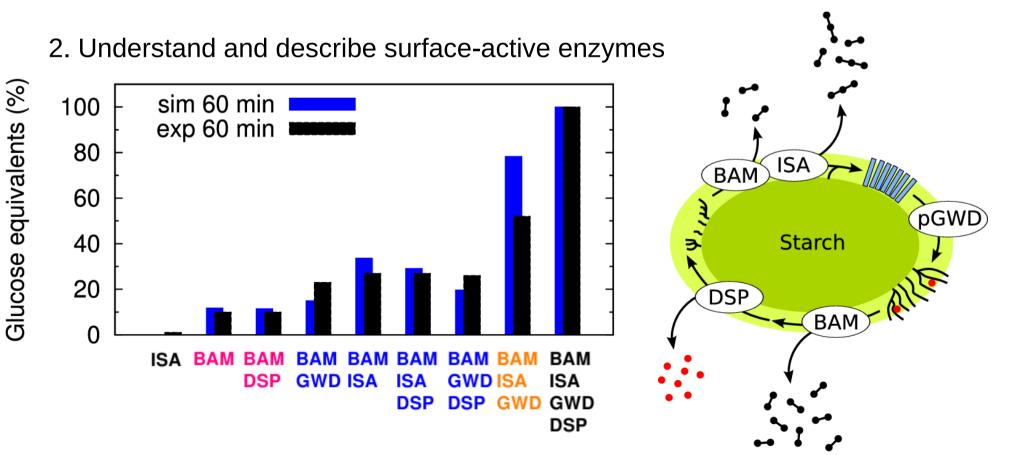
- in vitro kinetics of enzymes
- chain-length distributions for knockouts / synthetic in vitro-systems
- 2. Understand and describe surface-active enzymes

1. Understand and describe polymer-active enzymes

OK

Require more data:

- in vitro kinetics of enzymes
- chain-length distributions for knockouts / synthetic in vitro-systems



1. Understand and describe polymer-active enzymes

OK

Require more data:

- in vitro kinetics of enzymes
- chain-length distributions for knockouts / synthetic in vitro-systems
- 2. Understand and describe surface-active enzymes

ΟΚ

Require more data:

- in vitro kinetics of enzymes (difficult!)
- synthetic in-vitro systems with crystallised (ideal) starch
- time-resolved data!

3. Find the missing links!

1. Understand and describe polymer-active enzymes

OK

Require more data:

- in vitro kinetics of enzymes
- chain-length distributions for knockouts / synthetic in vitro-systems

2. Understand and describe surface-active enzymes

OK

Require more data:

- in vitro kinetics of enzymes (difficult!)
- synthetic in-vitro systems with crystallised (ideal) starch
- time-resolved data!
- 3. Find the missing links!

For example:

- formation of double helices (a-1,4-glucans)
- cooperation of biochemical and biophysical processes

Modelling 3D structure of polysaccharides

POLYS 2.0: An Open Source Software Package for Building Three-Dimensional Structures of Polysaccharides

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The next steps...

- Systematic *in vitro* characterisation of surface-active and polymer-active enzymes (Rob Field, JIC Norwich)
- Systematic experiments in yeast and combination of enzymes *in vitro* (Sam Zeeman, ETH Zurich)
- Combine existing modelling approaches
 (Oliver Ebenhöh, HHU Düsseldorf)

ERA-CAPS Project **DesignStarch**

Postdoc needed!

- Envisaged start: June 2015
- Goals:
 - synthesise starch in vitro and in yeast
 - model these processes
 - predict physico-chemical properties from biochemistry/biophysics
 - design starch!

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