





Mathematical Models of Plant Energy Metabolism

Towards synthetic starch

Oliver Ebenhöh



Why do we need mathematical models?

- Simplified representation of reality
- Reduction to the essentials

"Simplicity is the ultimate sophistication"

(Leonardo da Vinci)

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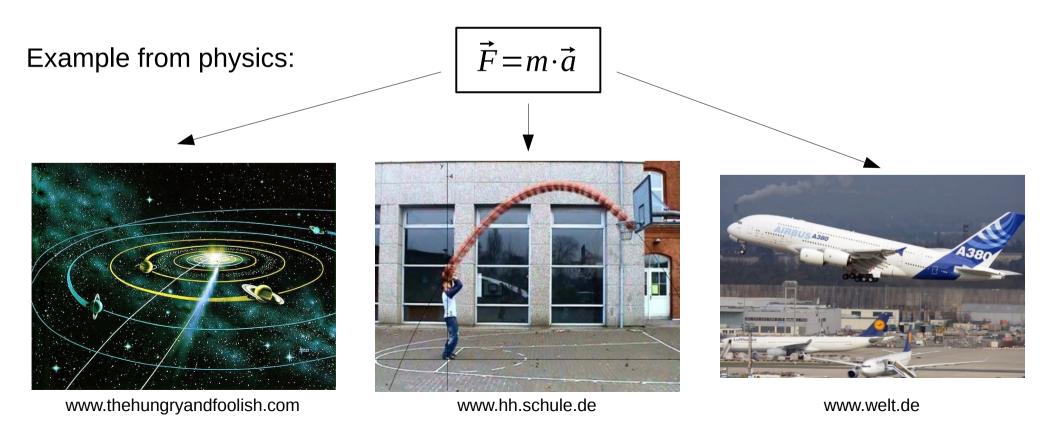
Why do we need mathematical models?

- Simplified representation of reality
- Reduction to the essentials

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Models help to discover general principles!







How does one find principles (theory building)?

$$\overrightarrow{F} = m \cdot \overrightarrow{a}$$



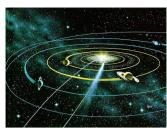


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 $\vec{F} = m \cdot \vec{a}$

Intuition

Every model is a small step on this path



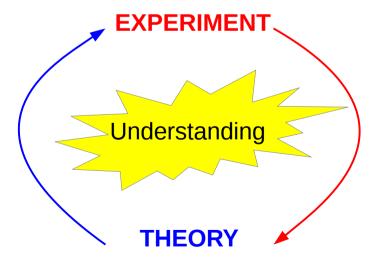
How does one find principles (theory building)?

$$\vec{F} = m \cdot \vec{a}$$

Intuition

Every model is a small step on this path

- Model predictions / new hypotheses
- Suggestions for new experiments
- Improvement of experimental design



- Initial model formulation
- Confirmation / falsification of predictions
- New model assumptions

The Systems biology principle

What's special about plants?

- 1.Photosynthesis
- 2.Can't run away!

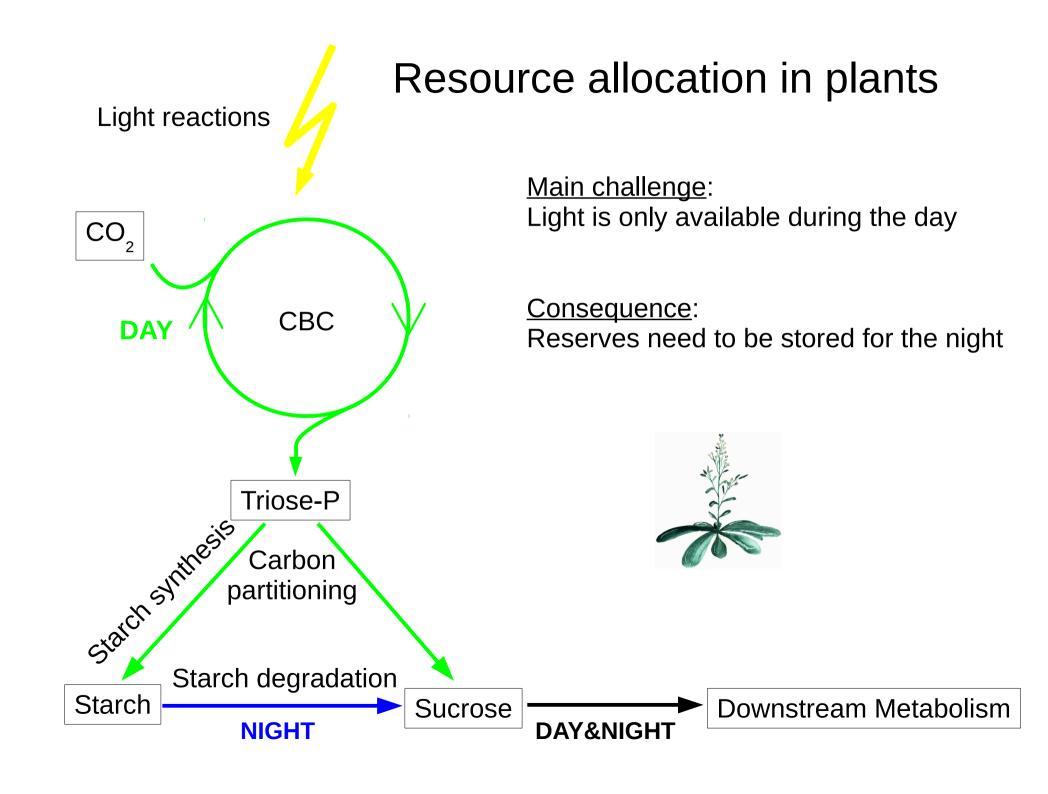


Experts in chemical warfare!

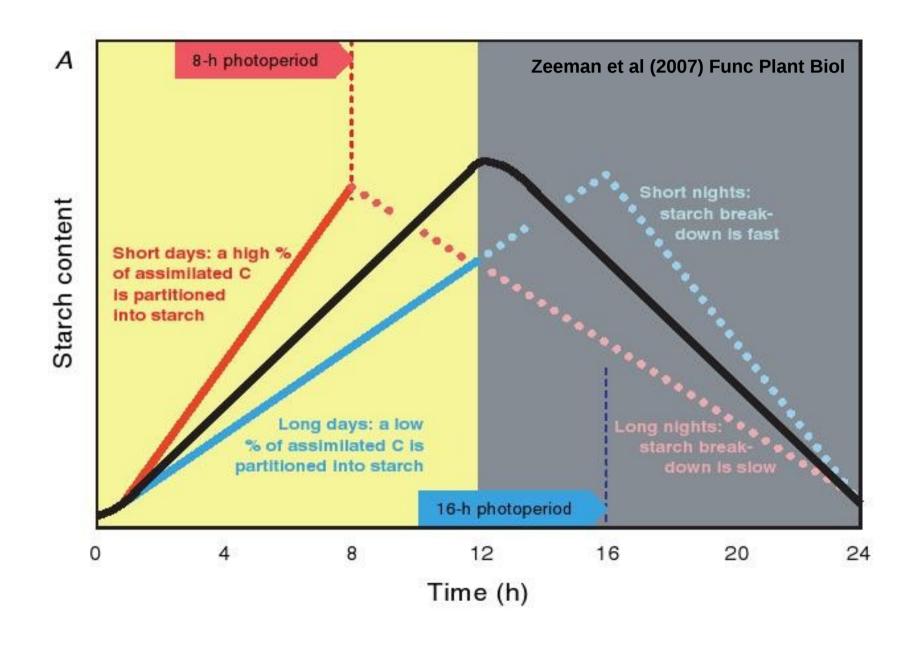
Estimated > 200,000 secondary metabolites!



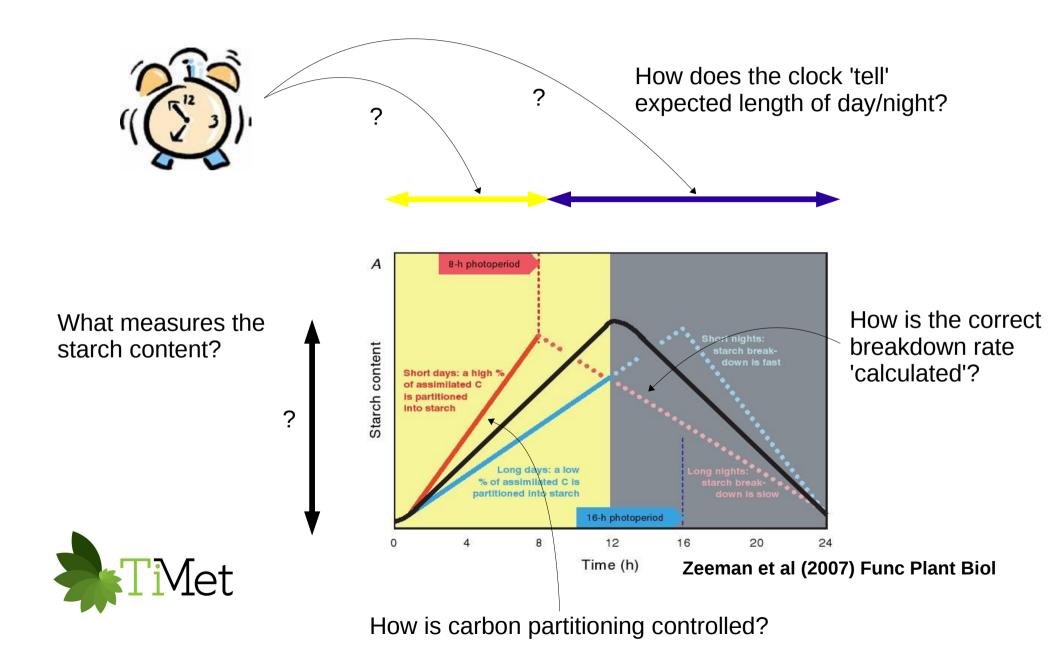
(commons.wikimedia.org)



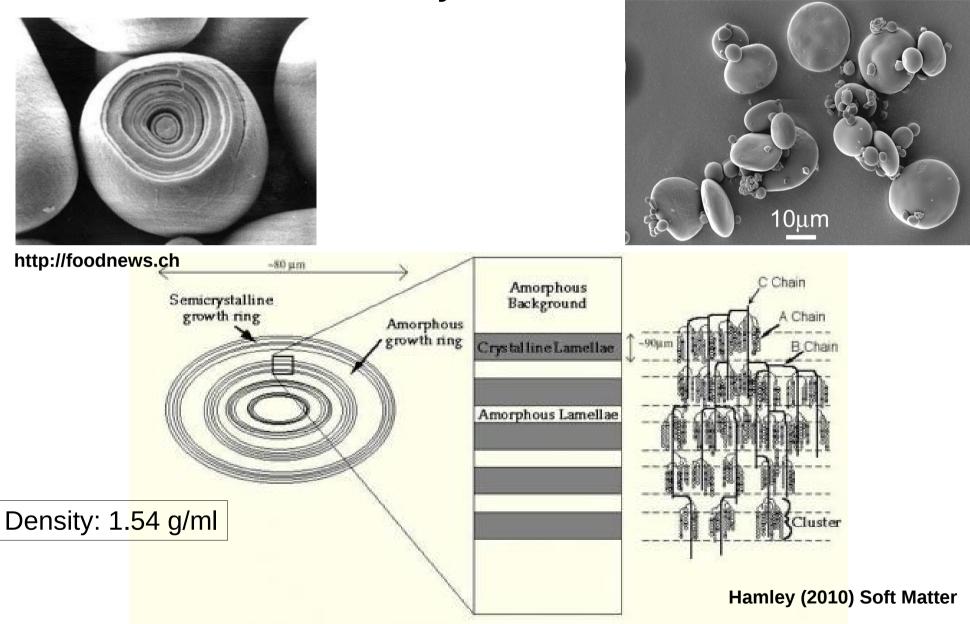
The diurnal turnover of starch



Open questions



Why starch?



The structure of starch allows for an extremely high energy storage density

Alternatives

energy content (kJ/g)

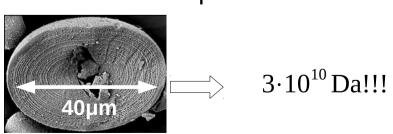
Carbohydrates	17
Lipids	38
Proteins	17
Alcohol	30

We (animals and fungi) predominantly use glycogen



big molecule (up to 10 MDa)

still small compared to starch



http://swissplantscienceweb.ch

Possible advantages of starch

- low osmolarity
- large size
- high density

Alternatives

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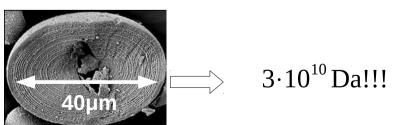
trade-off between storage density and rapid mobilization

big molecule (up to 10 MDa)

Possible advantages of starch

- low osmolarity
- large size
- high density

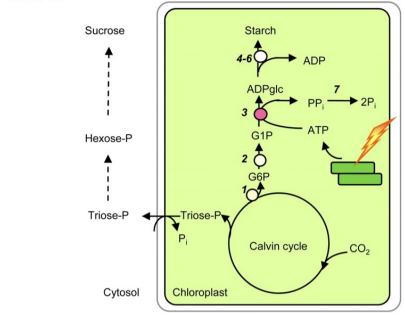
still small compared to starch

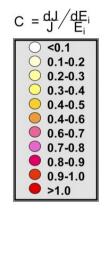


optimised for storage density, slower deployment

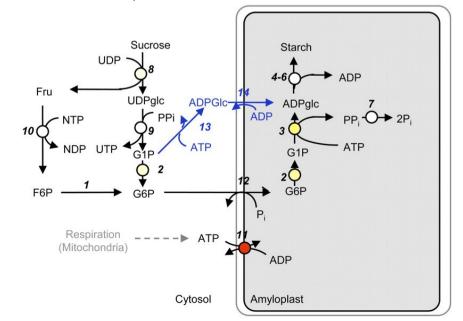
How is starch made?

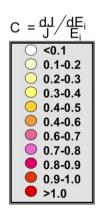
A Leaves





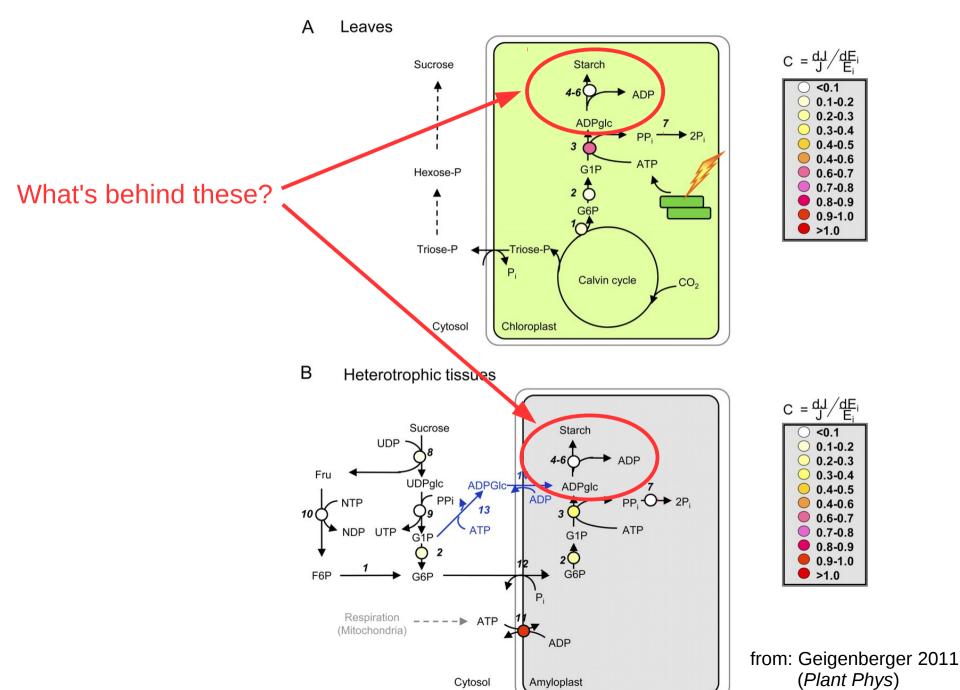
B Heterotrophic tissues



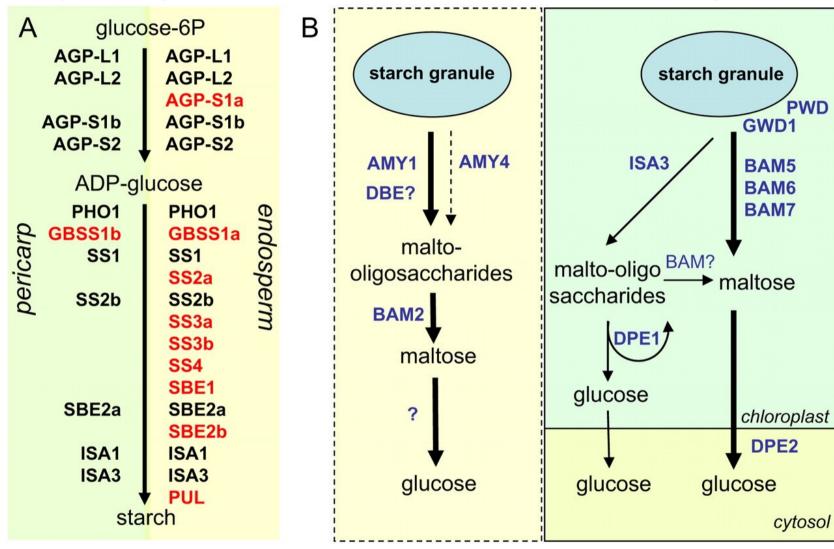


from: Geigenberger 2011 (*Plant Phys*)

How is starch made?



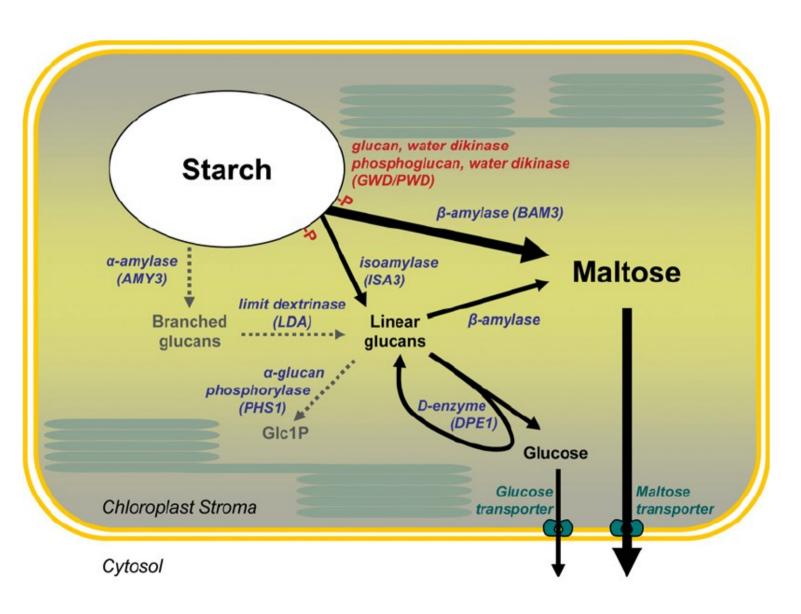
Many enzymes are involved in starch synthesis



- starch synthases
- branching enzymes
- phosphorylases
- isoamylases

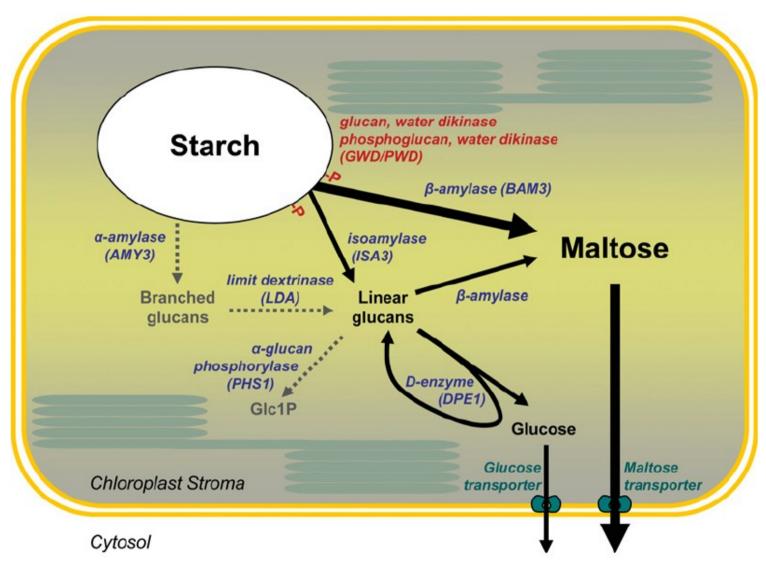
from: Radchuk et al 2009 (Plant Phys)

...and starch breakdown



from: Zeeman et al, 2007, Biochem J

...and starch breakdown



Many enzymes

are surface-active

or

act on polymers

J

hard to describe with traditional modelling approaches

from: Zeeman et al, 2007, Biochem J

Challenges / Topics of lecture

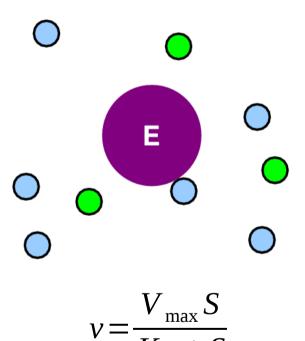
- 1. Surface-active enzymes
- 2. Polymer-active enzymes
- 3. Timing of starch metabolism

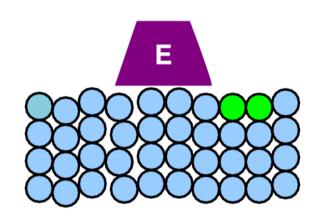
1. Surface-active enzymes

Rate laws for surfactive enzymes

dissolved substrate

aggregated substrate (with interfacial reaction space)

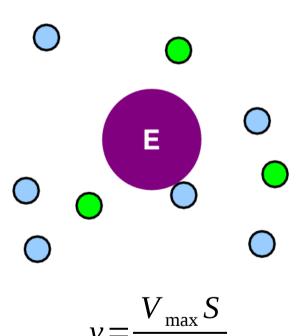




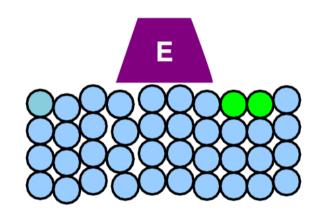
$$v = f(?)$$

Rate laws for surfactive enzymes

dissolved substrate



aggregated substrate (with interfacial reaction space)



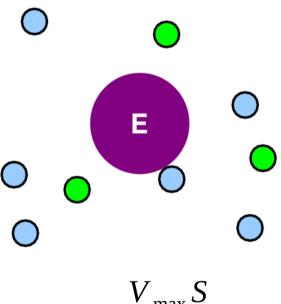
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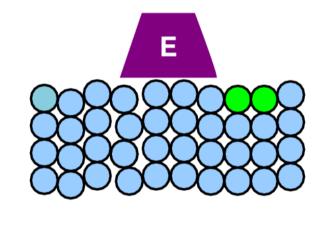
Reaction space confined to 2D!

Rate laws for surfactive enzymes

dissolved substrate

aggregated substrate (with interfacial reaction space)





$$v = \frac{V_{\text{max}}S}{K_M + S}$$

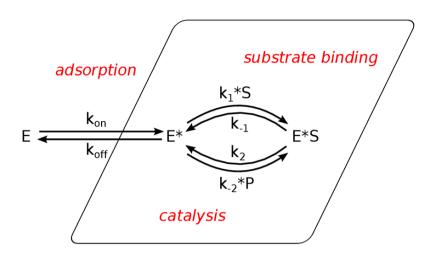
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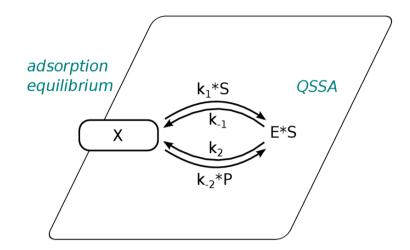
Reaction space confined to 2D!

Implications! - Fundamental differences to the classical case in solution:

- Relative activity dependent on enzyme concentration (jamming)
- Rate not independent on presence of other enzyme species! (competition)

Derivation of a generic surfactive rate-law





The adsorption equilibrium

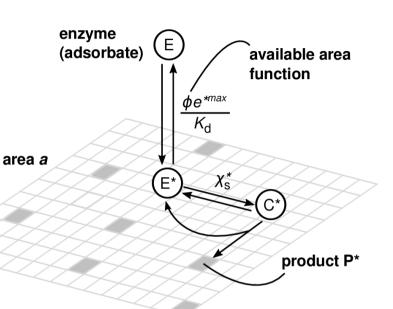
The Langmuir isotherm (a concept from surface physics)

Adsorption coverage (surface concentration):

$$\theta_E = \frac{n(E)}{n(E)_{\text{max}}} = \frac{n(E)}{E_{\text{max}} \cdot S}$$

Adsorption rate: $r_a \propto c(E) \cdot (1 - \theta_E)$

Desorption rate: $r_d \propto \theta_E$



The adsorption equilibrium

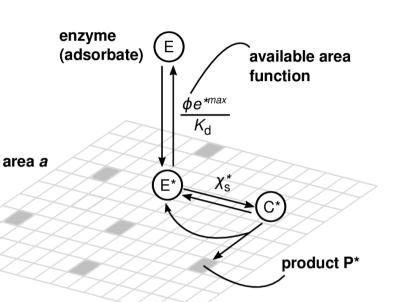
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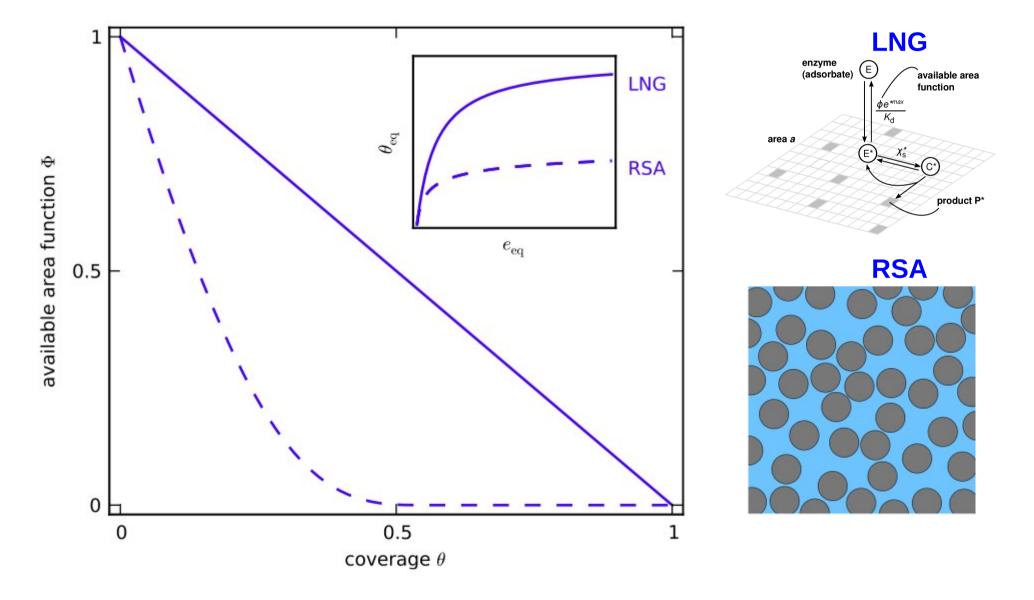
Desorption rate: $r_d \propto \theta_E$



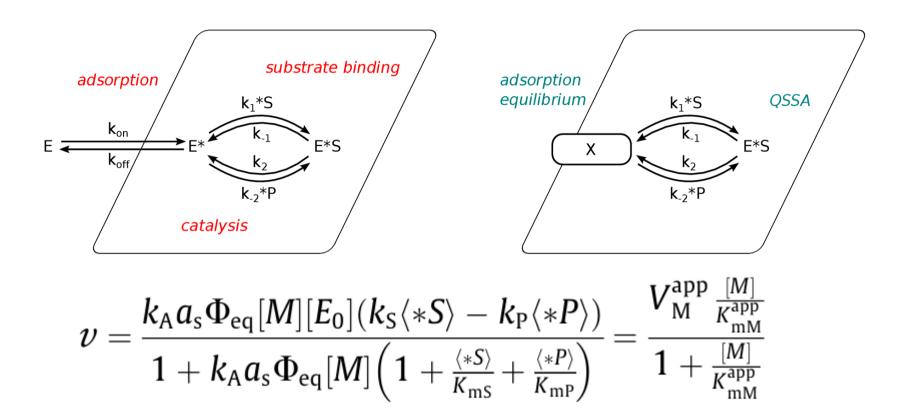
Available area function

The adsorption equilibrium

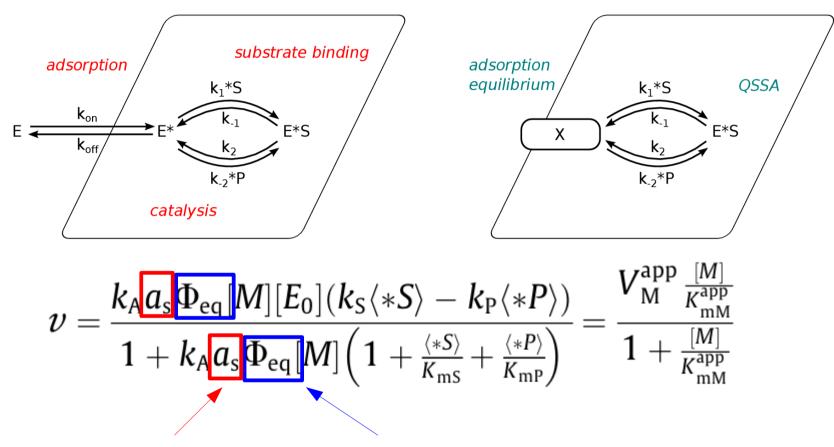
Other adsorption models can give quite different results:



Derivation of a generic surfactive rate-law



Derivation of a generic surfactive rate-law



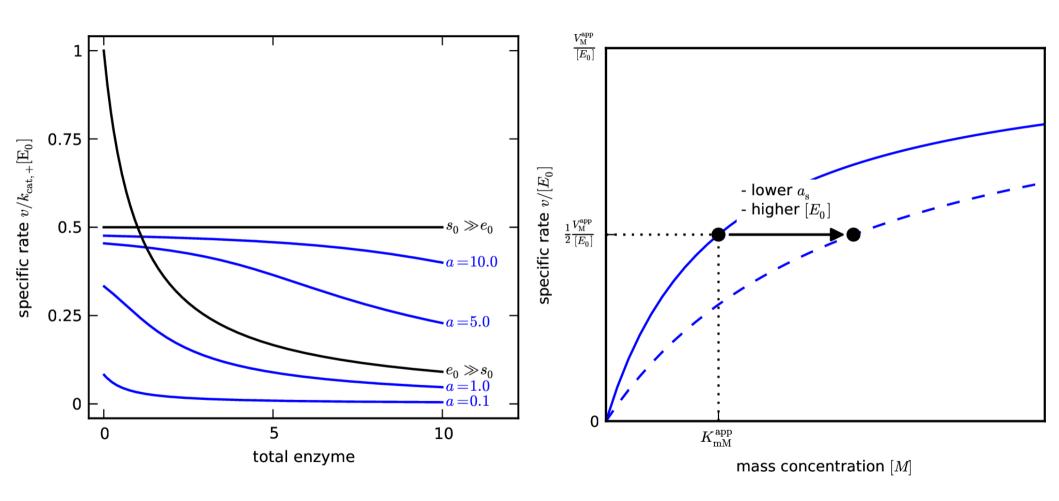
specific surface area

available area function

"few big objects behave different to many small objects"

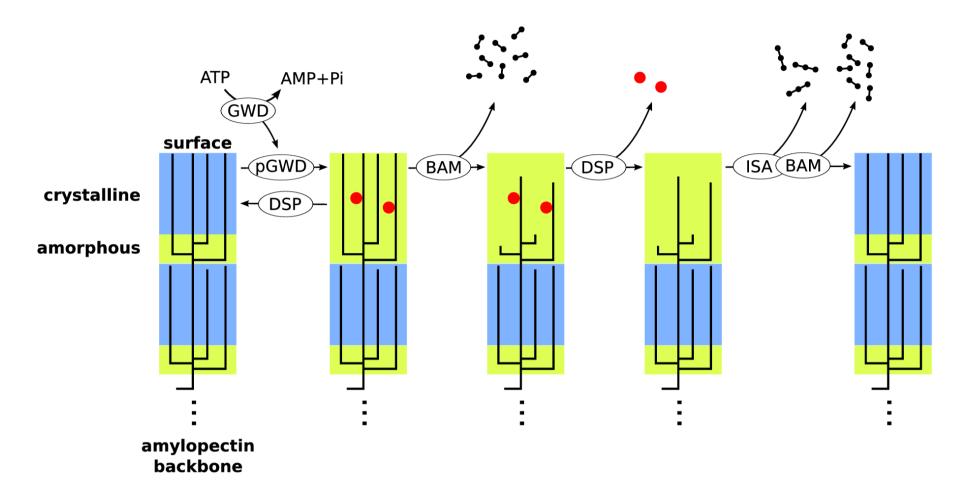
"many enzymes (also others) jam the surface"

Consequences for experimental design



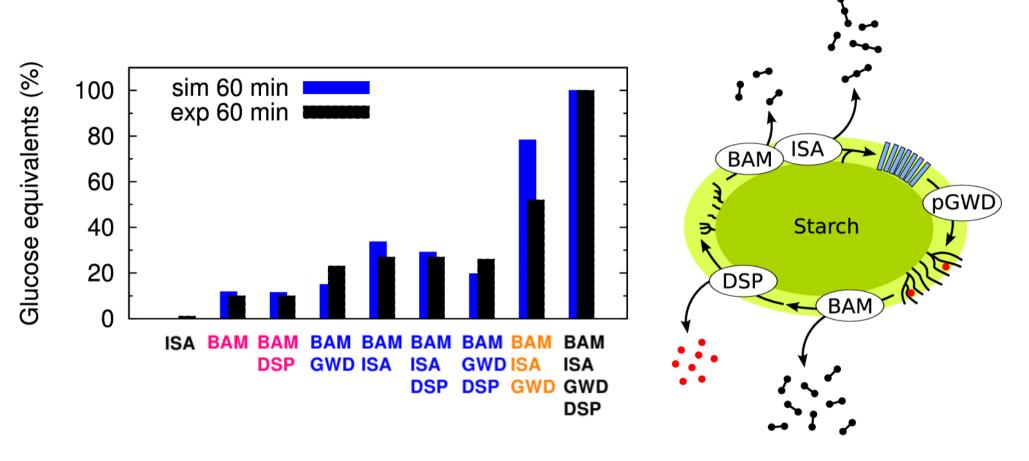
mass alone is insufficient!

A kinetic model of starch surface attack



- Disruption of crystalline surface by phosphorylation allows access for BAM and ISA
- Dephosphorylation by DSP enables further degradation

Simulations compared to experiment

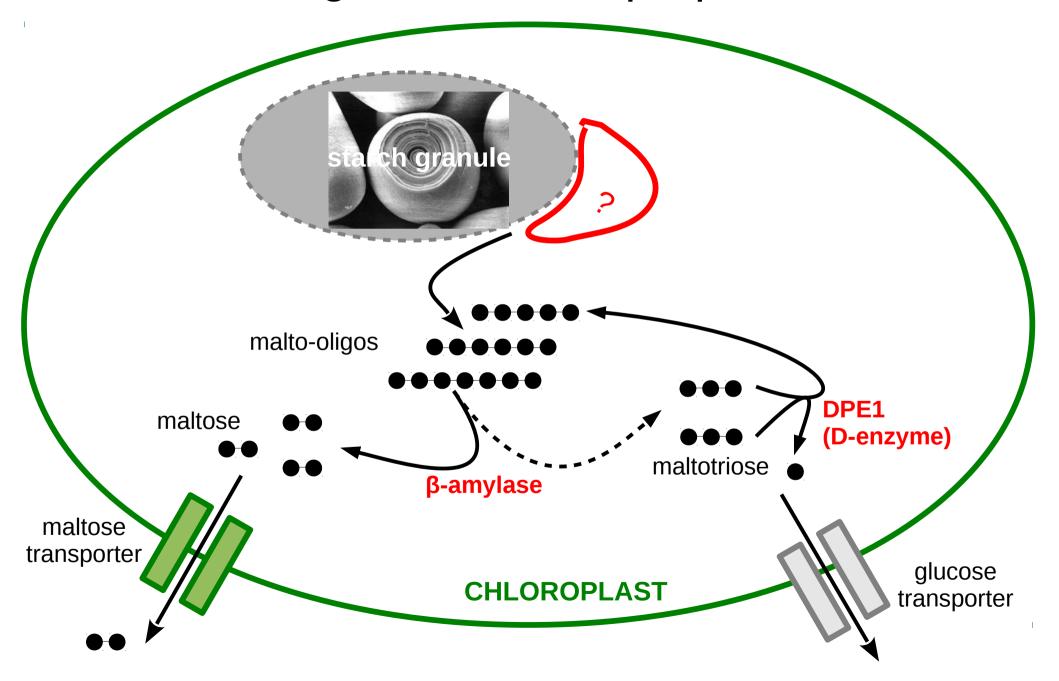


Good agreement with data from Kötting et al (2009) Plant Cell

But: only one time point!

2. Polymer Biochemistry

Starch degradation - disproportionation



Disproportionating enzymes (D-enzymes)

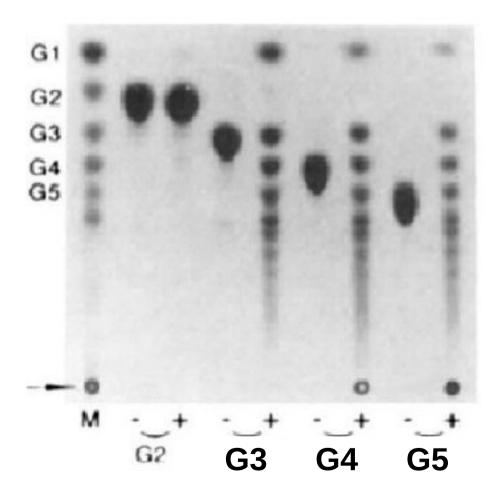
DPE1

catalyses 2 maltotriose — maltopentaose + glucose

EC: 2.4.1.25

 $G3+G3 \longrightarrow G5+G1$

but not only!



DPE1 produces a set of glucans of different length in in vitro assays.

(Takaha et al., JBC 1993)

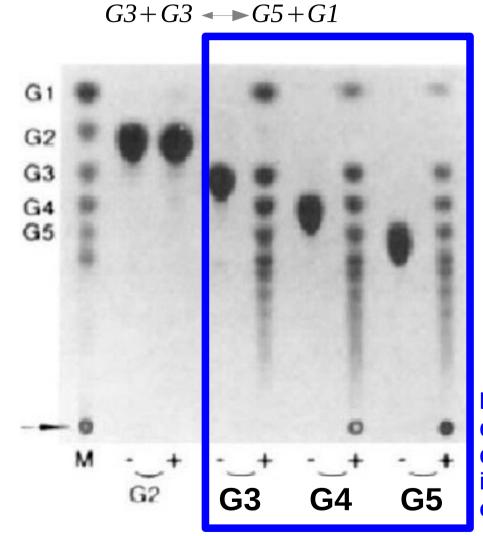
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Equilibrium distribution depends on initial conditions!

(Takaha et al., JBC 1993)

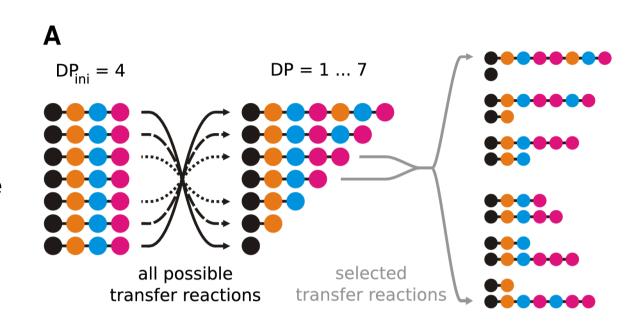


Disproportionating enzymes (D-enzymes)

DPE1

EC: 2.4.1.25

Disproportionating Enzyme randomises DPs



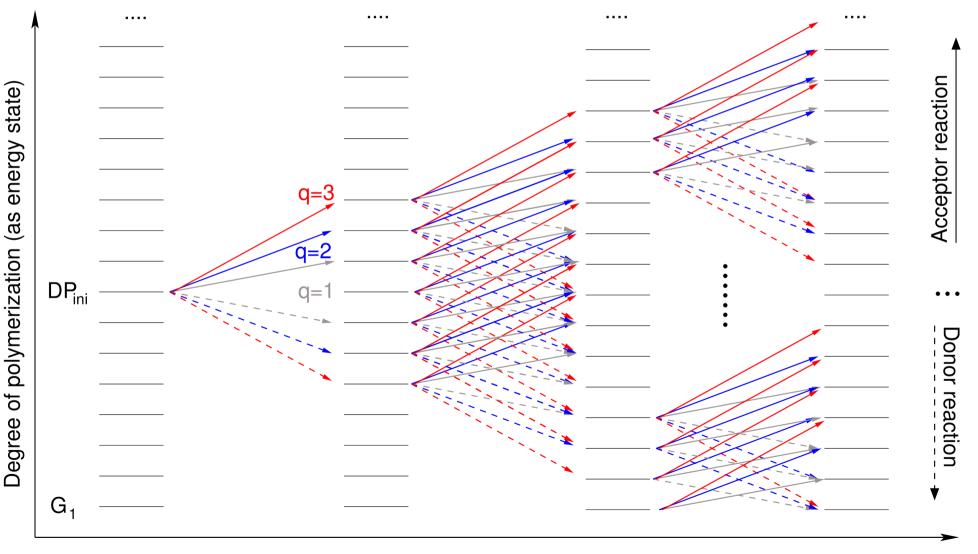
transfers glucosyl residues from one glucan to another: $G_n+G_m - G_{n-q}+G_{m+q}$ reaction must proceed towards a smaller Gibbs free energy : $\Delta G = \Delta H - T \Delta S < 0$ energy neutral (enthalpy of α -1,4-bond hydrolysis independent on position): $\Delta H = 0$ (Goldberg et al, 1992)



DPE1 maximises the entropy of the polydisperse reactant mixture

The thermodynamic picture

- Different DPs are interpreted as different energy states (energy of formation)
- Enzymes mediate transitions between these states



Polydisperse mixtures as statistical ensembles

 X_i : molar fraction of glucans with length i corresponds to occupation number of state i

The distribution X_i fully characterises the polydisperse reactant mixture

The entropy of the statistical ensemble is $S = -\sum x_k \ln x_k$

Equilibrium is determined by maximal entropy:

$$S = -\sum x_k \ln x_k \to \max!$$

Maximum entropy principle under constraint that #bonds and #molecules is conserved!

conservation of #molecules: $\sum x_k = 1$

conservation of #bonds: $\sum k \cdot x_k = b$

determined by initially applied mixture of maltodextrins

Solution using Lagrangian multipliers: Necessary conditions are given by

$$\frac{\partial L}{\partial x_k} = 0 \quad \text{with} \quad L(x_k; \alpha, \beta) = \sum_k x_k \ln(x_k) + \alpha \left(\sum_k x_k - 1\right) + \beta \left(\sum_k k \cdot x_k - b\right)$$

$$\Leftrightarrow \ln(x_k) + 1 + \alpha + k \beta = 0 \text{ for all } k$$

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 for all k

Analogy to statistical physics! There, $\beta = \frac{1}{k_B \cdot T}$

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 $\Leftrightarrow \ln(x_k) + 1 + \alpha + k \beta = 0$ for all k

$$x_k = \frac{1}{Z} e^{-k\beta} \text{ with } Z = \sum_k e^{-k\beta}$$

Analogy to statistical physics! There, $\beta = \frac{1}{k_B \cdot T}$

Calculation of
$$\beta$$
: $-\frac{1}{Z}\frac{\partial Z}{\partial \beta} = b \iff \beta = \ln \frac{b+1}{b}$

Solution using Lagrangian multipliers: Necessary conditions are given by

$$\frac{\partial L}{\partial x_k} = 0 \quad \text{with} \quad L(x_k; \alpha, \beta) = \sum_k x_k \ln(x_k) + \alpha \left(\sum_k x_k - 1\right) + \beta \left(\sum_k k \cdot x_k - b\right)$$

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Analogy to statistical physics! There, $\beta = \frac{1}{k_B \cdot T}$

Calculation of
$$\beta$$
: $-\frac{1}{Z}\frac{\partial Z}{\partial \beta} = b \iff \beta = \ln \frac{b+1}{b}$

Maximal entropy in equilibrium: $S_{max} = (b+1)\ln(b+1) - b\ln b$

$$S = -\sum x_k \ln x_k \to \max!$$

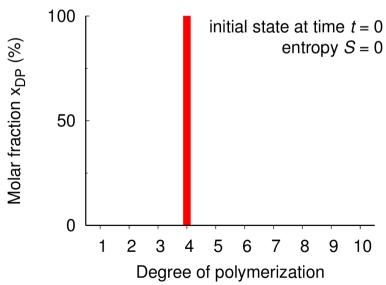
conservation of #molecules: $\sum x_k = 1$

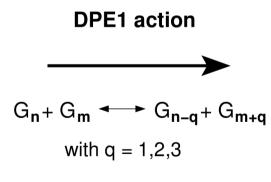
conservation of #bonds: $\sum k \cdot x_k = DP_{ini} - 1$

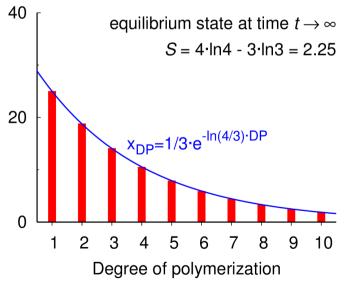


$$x_i = \frac{1}{Z} e^{-\beta E_i}, \beta = \ln \frac{DP_{ini}}{DP_{ini} - 1}$$

predicts





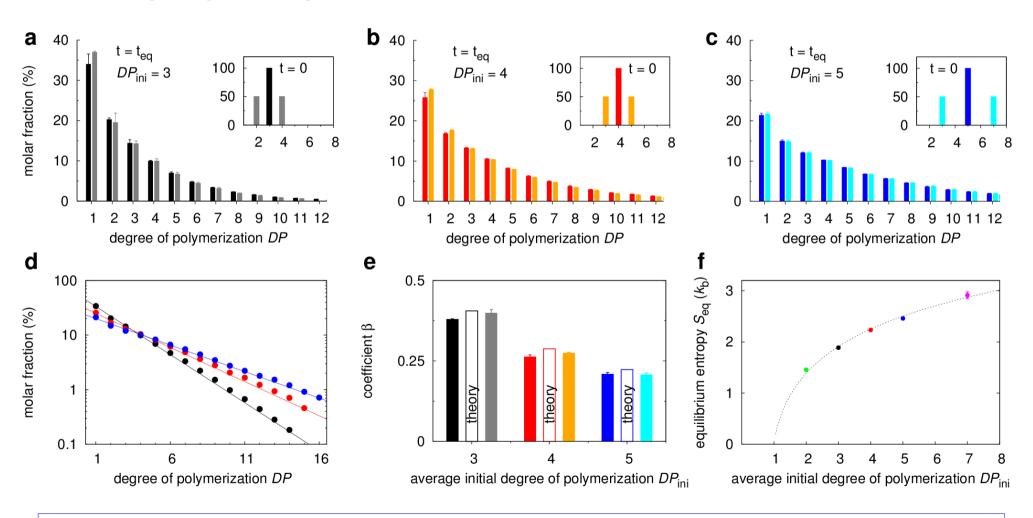


An instance of the 2nd law of TD!

DPE1 is entropy driven

Experiments with Martin Steup, University of Potsdam

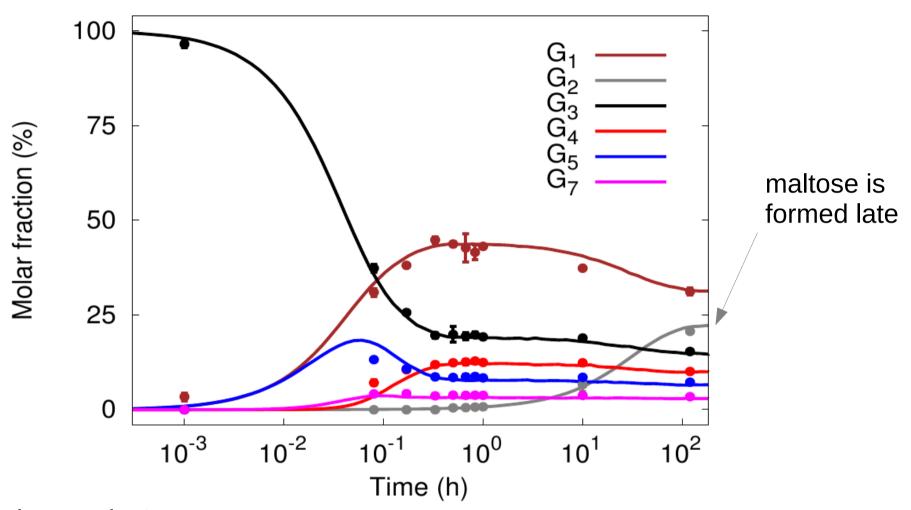
method: capillary electrophoresis



 β is a generalisation of the equilibrium constant for polydisperse mixtures

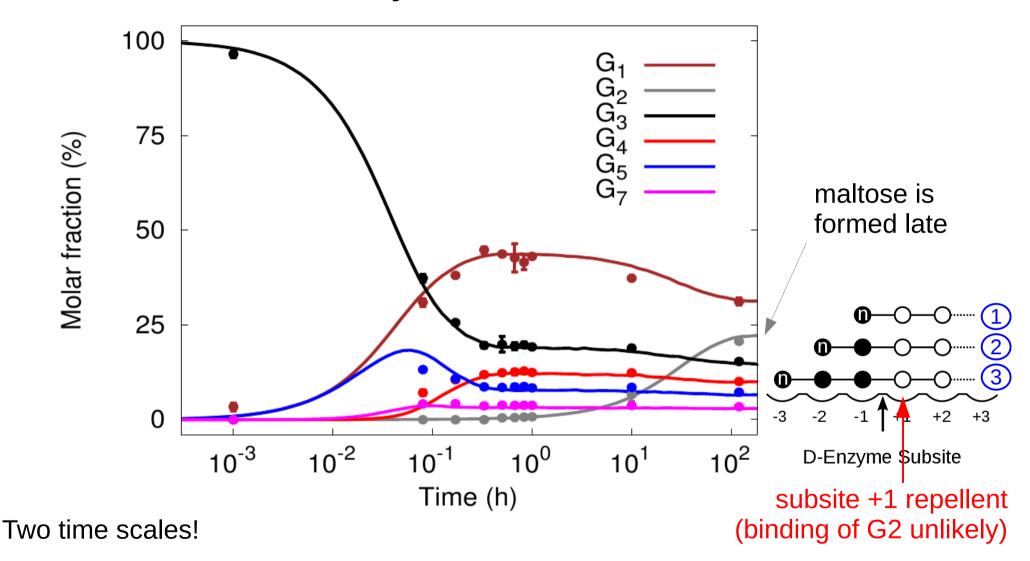
(Kartal et al, 2011, Mol Syst Biol)

The dynamics of DPE1



Two time scales!

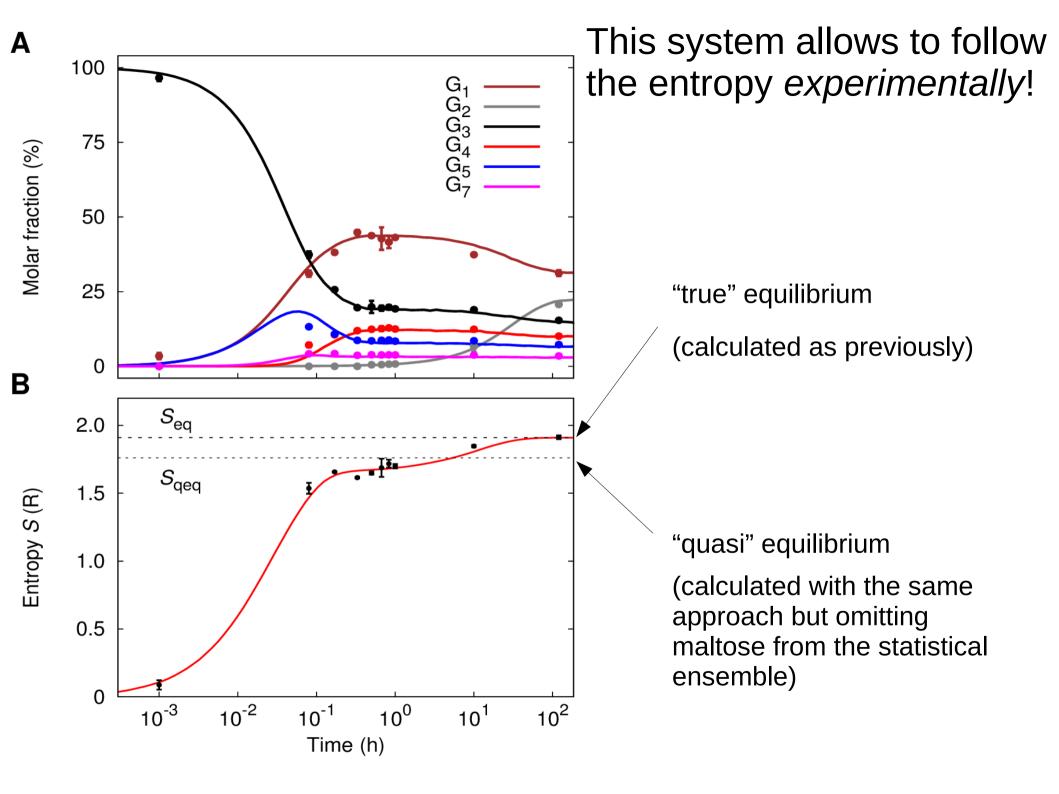
The dynamics of DPE1

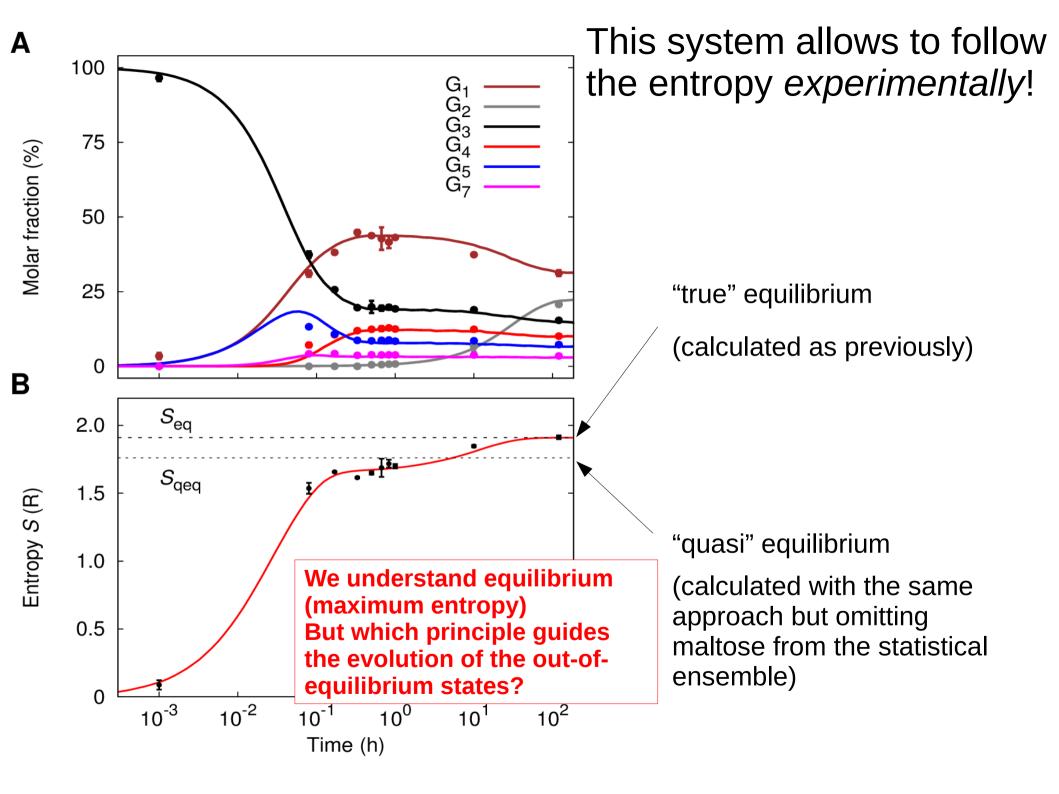


ratio 1:800

The simulations used 3 parameters:

- maximal turnover
- affinity for positional isomer 1
- affinities for positional isomers 2 and 3





Theory is also confirmed by DPE2

DPF2 vs DPF1

- transfers single glucosyl residues
- G2 only used as donor
- G3 only used as acceptor

Generic reaction catalysed:

$$G_n + G_1 \longrightarrow G_{n-1} + G_2$$

Entropic principle:

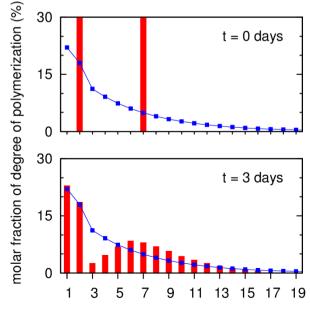
$$S = -\sum_{k} x_{k} \ln x_{k} \to \max$$

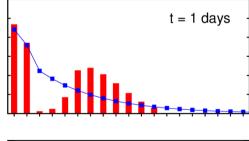
with one additional side constraint

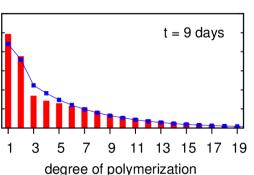
$$x_1 + x_2 = m = \text{const.}$$
 (and $\sum x_k = 1$; $\sum k \cdot x_k = b$)

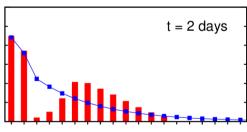
$$\Rightarrow x_i = \frac{1}{Z} e^{-\beta E_i} \text{ for } i \ge 3 \text{ where } \beta \text{ fulfils } b - 2(1-m) = m \cdot \frac{e^{-\beta}}{1 + e^{-\beta}} + (1-m) \cdot \frac{e^{-\beta}}{1 - e^{-\beta}}$$

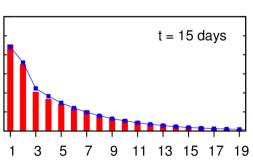
Experiment Theory











Theory is also confirmed by DPE2

DPF2 vs DPF1

- transfers single glucosyl residues
- G2 only used as donor
- G3 only used as acceptor

Generic reaction catalysed:

$$G_n + G_1 \longrightarrow G_{n-1} + G_2$$

$$\Rightarrow x_i = \frac{1}{2}e^{-\beta E_i}$$
 for $i \ge 3$ where β fulfils

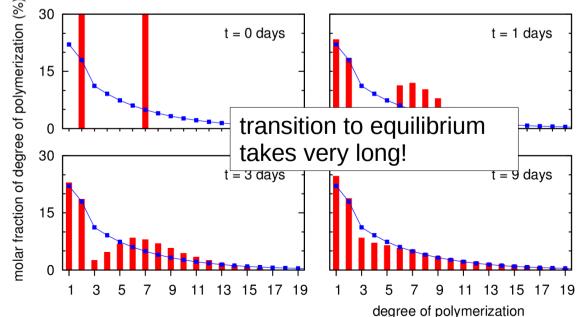
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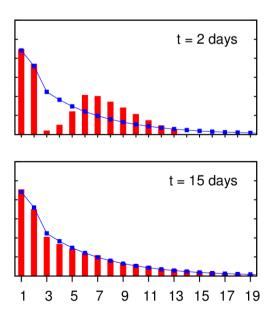
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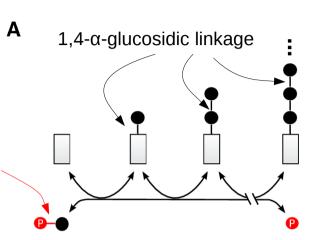
Generalisation to non-zero enthalpy changes

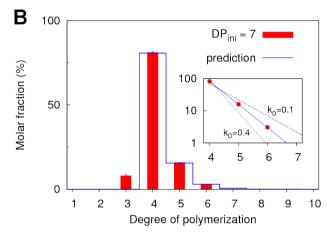
Phosphorylase (cPho):

$$P_i + G_n \longrightarrow G1P + G_{n-1}$$

 $\Delta H \neq 0!$

phosphoester bond





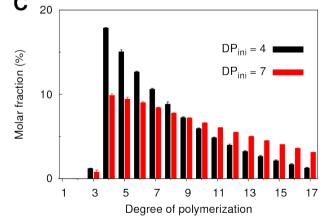
Generalisation by including energetic and entropic contributions:

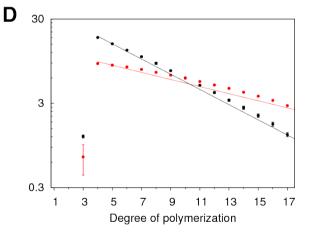
$$G = G^f - T \cdot S_{mix} \to \min!$$

Gibbs energy of formation

mixing entropy:

$$S_{mix} = -R \sum_{k=1}^{\infty} x_k \ln x_k$$



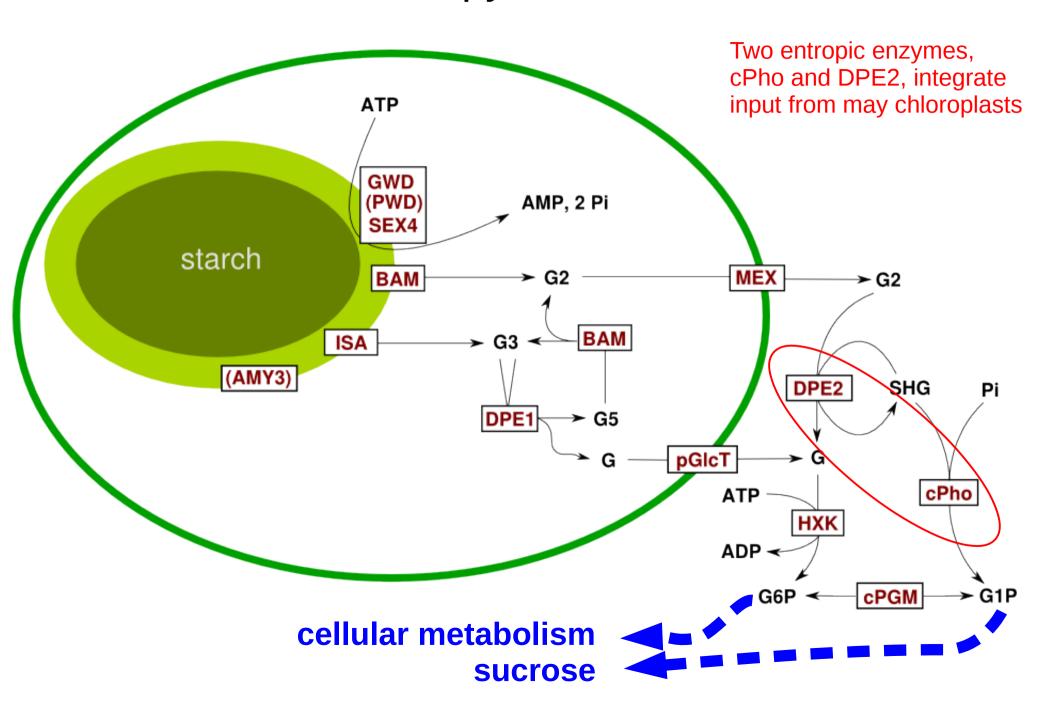


Prediction: Similar pattern as for DPE2

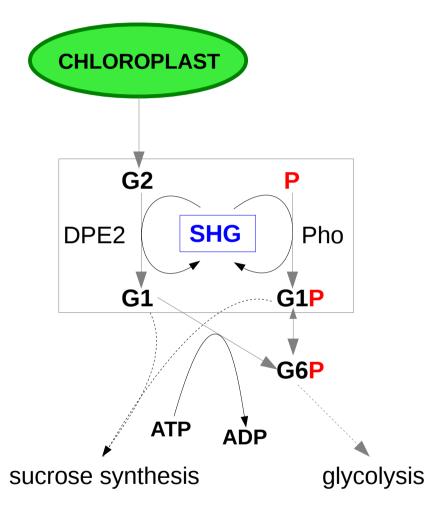
Experimentally confirmed.

(Kartal et al, Supp to MSB 2011; Ebenhöh et al, Proc 5th ESCEC 2013)

An entropy-driven buffer



What is the role of the SHG pool?

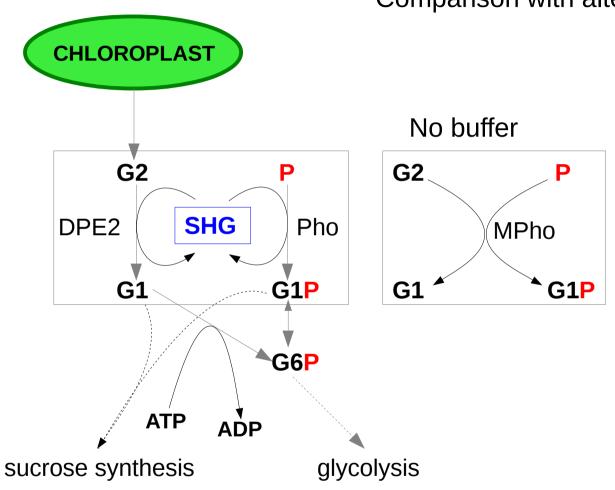


Two 'entropic' enzymes mediate the turnover of a polydisperse pool

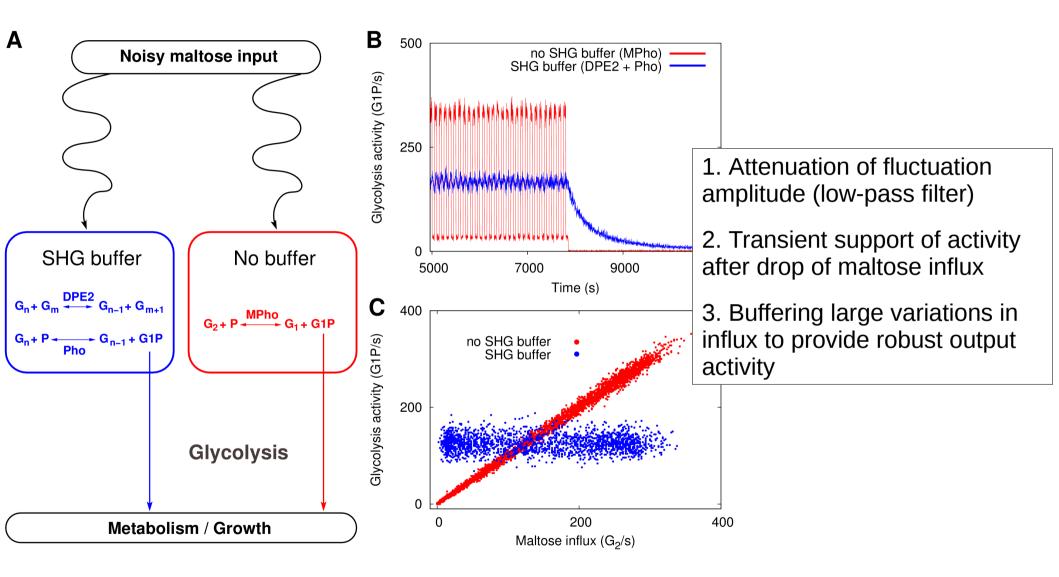
What is the advantage over other hypothetical systems?

What is the role of the SHG pool?

Comparison with alternative

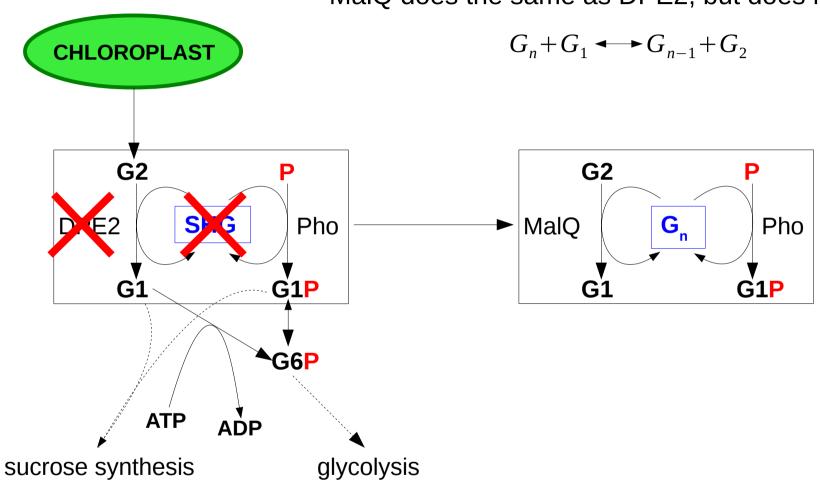


Polydisperse SHG pools increases robustness in vivo



Replacing DPE2 by MalQ

MalQ does the same as DPE2, but does not use SHG



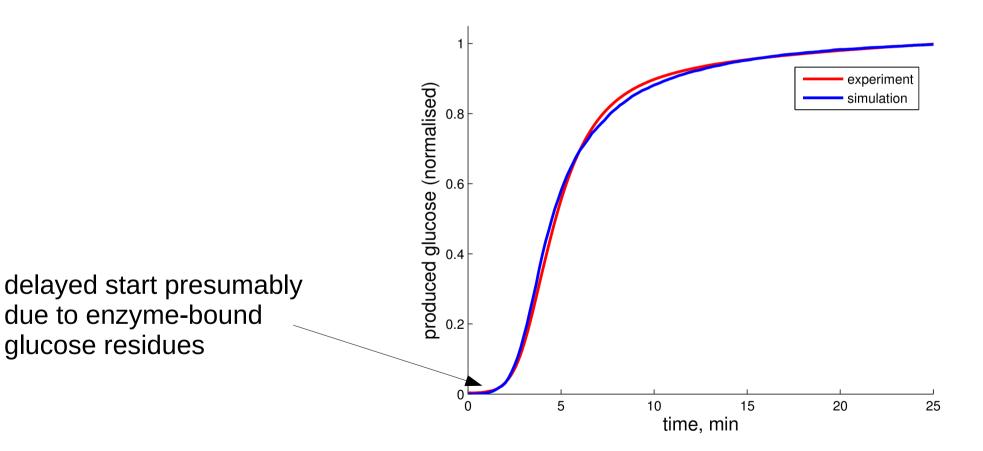
Simulating MalQ in vitro kinetics

In vitro system: DPE1 + HXK

Incubation with G₂ only!

$$G_n + G_1 \longrightarrow G_{n-1} + G_2 \quad n \neq 3$$

 $G_1 \longrightarrow \emptyset$



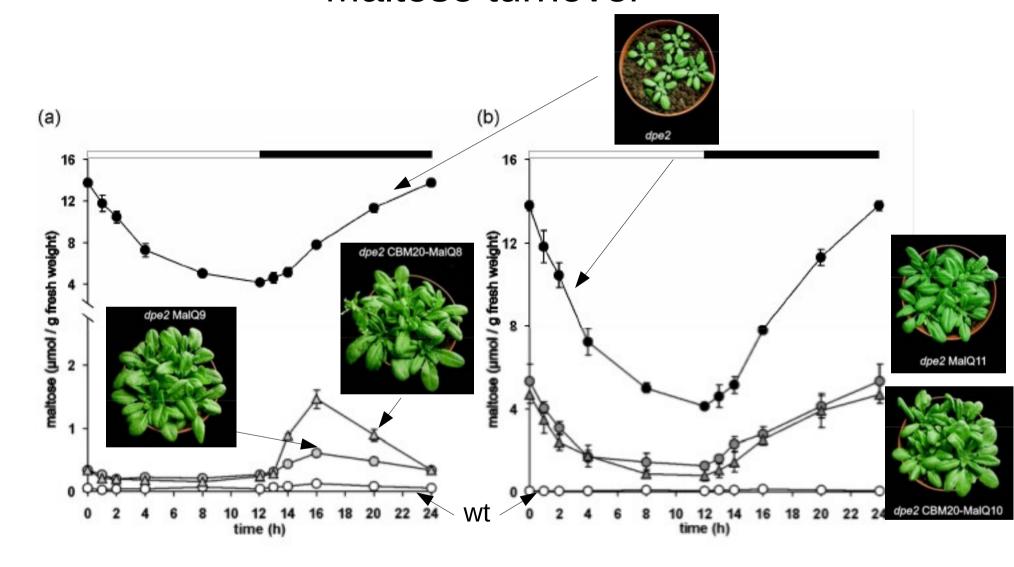
Moderate growth phenotype



(Julia Smirnova, PhD thesis; Ruzanski et al, JBC 2013)

complemented plants grow OK!

Maltose turnover

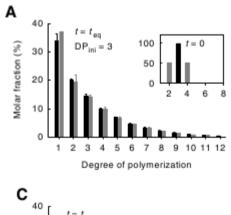


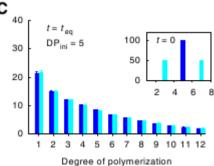
Where else do find entropic enzymes?

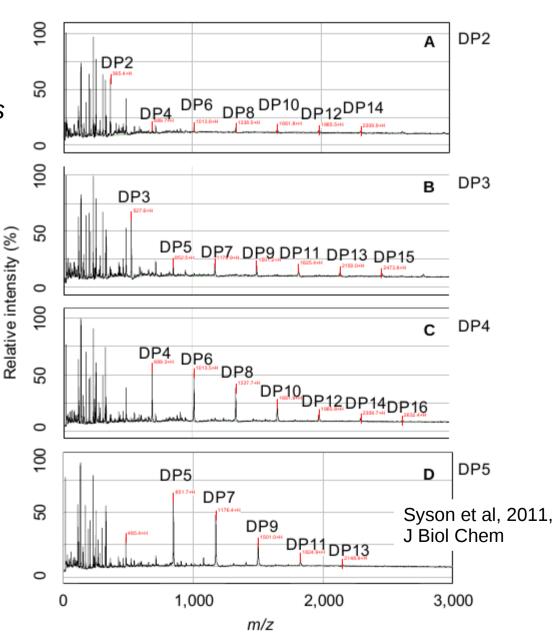
...for example

Maltosyltransferases in *Streptomyces*

"Acceptor specificity" can be explained by entropic principles

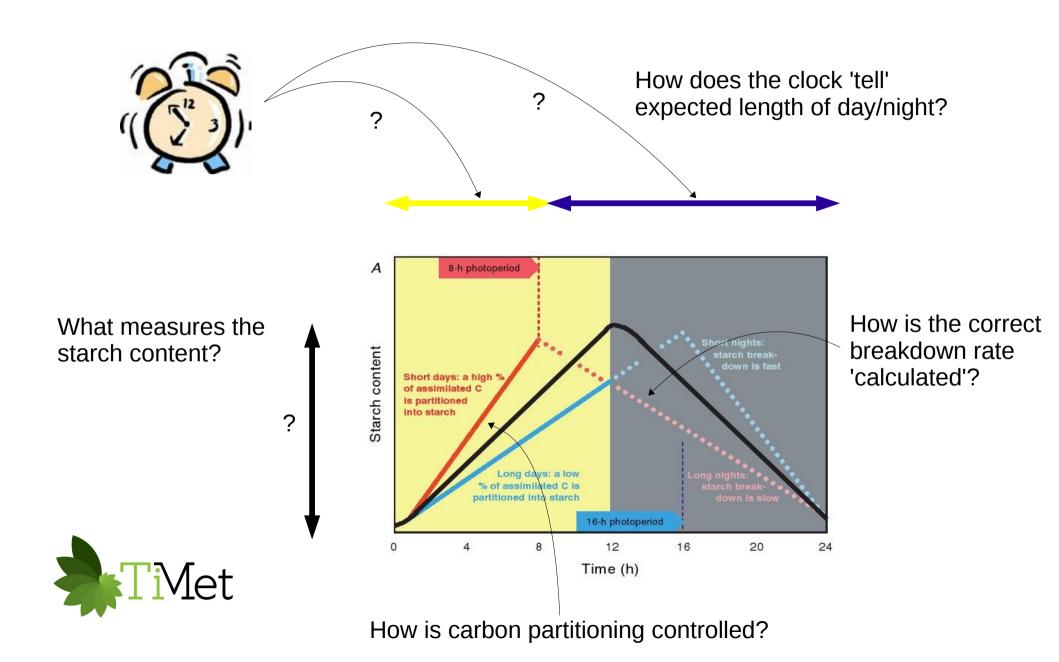






3. Timing of Metabolism

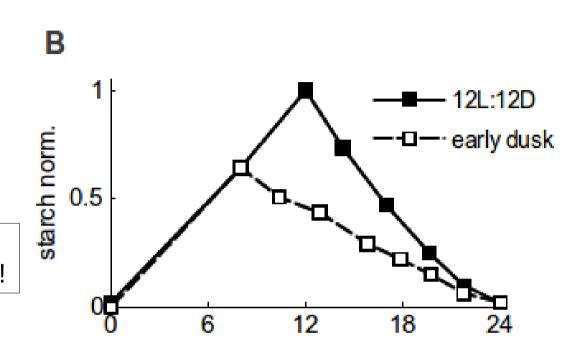
Open questions



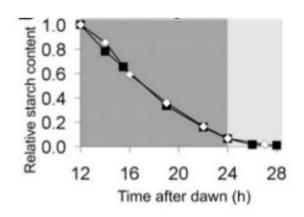
...even more mysteries...

The 'early dusk' experiment by Alexander Graf, (Graf et al 2010, PNAS)

Even when 'surprised' by a 4 hour shorter day, plants 'know' what to do!



The circadian clock is apparently important, because:



Plants cannot adapt to T-cycles different than 24h!

Building a mathematical model

Known:

- Metabolism
- Circadian clock



- Regulation of starch synthesis
- Regulation of starch breakdown
- How is starch content measured?



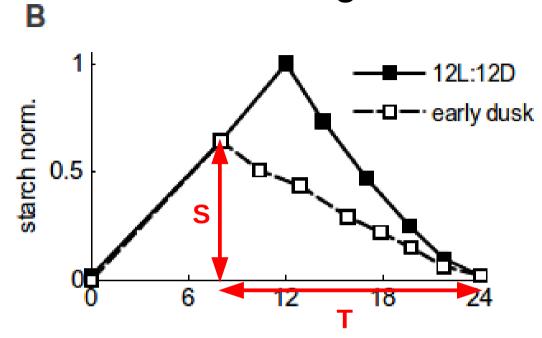
Alexandra Pokhilko

Challenges:

- 1. The model must combine known systems with plausible, but hypothesised regulatory mechanisms
- 2. To keep the model tractable, we need to find a compromise between detailedness and simplification

Seaton et al, 2013, Roy Soc Interface; Pokhilko et al, 2014, Mol BioSyst; Pokhilko et al, 2015, Roy Soc Interface

How to regulate starch degradation?



Arithmetic division

$$v = \frac{S}{T}$$

Simplest solution:

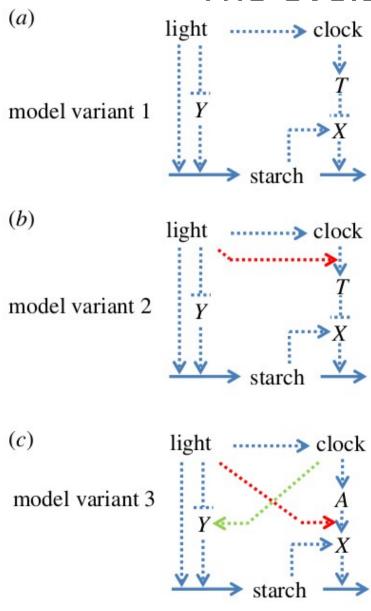
Auxiliary compound X (e.g. active form of starch degrading enzyme):

$$\frac{dX}{dt} = k_1 S - k_2 X T$$

Rapid activation/deactivation: $\frac{dX}{dt} = 0 \iff X = \frac{k_1}{k_2} \cdot \frac{S}{T}$

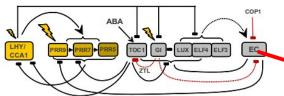
Scaldione et al (2013), eLife: Arabidopsis plants perform arithmetic division to prevent starvation at night

The evolution of a model

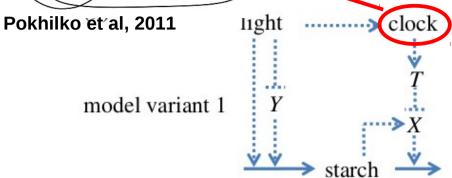


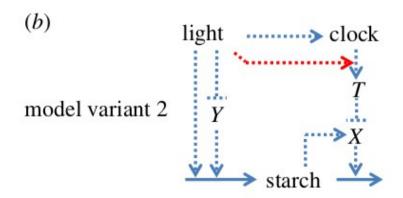
In Seaton et al, 2013:

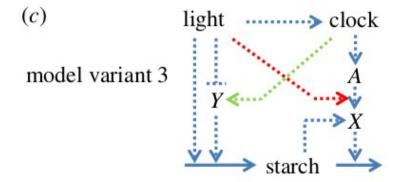
Testing basic regulatory mechanisms



The evolution of a model





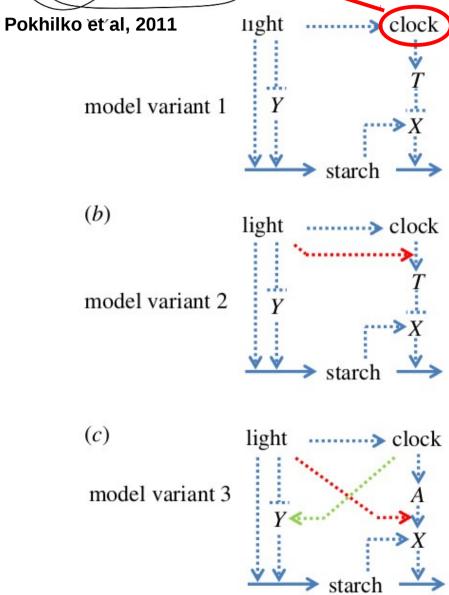


In Seaton et al, 2013:

- Testing basic regulatory mechanisms
- Replacing 'clock' by a detailed model

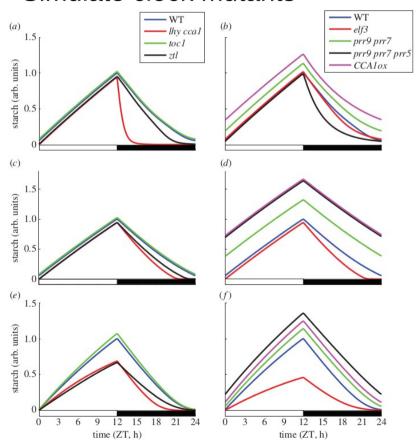
ABA LHY CCA1 PRRS PRRS PRRS TOCS GI LUX ELF4 ELF3 EC ZTL

The evolution of a model

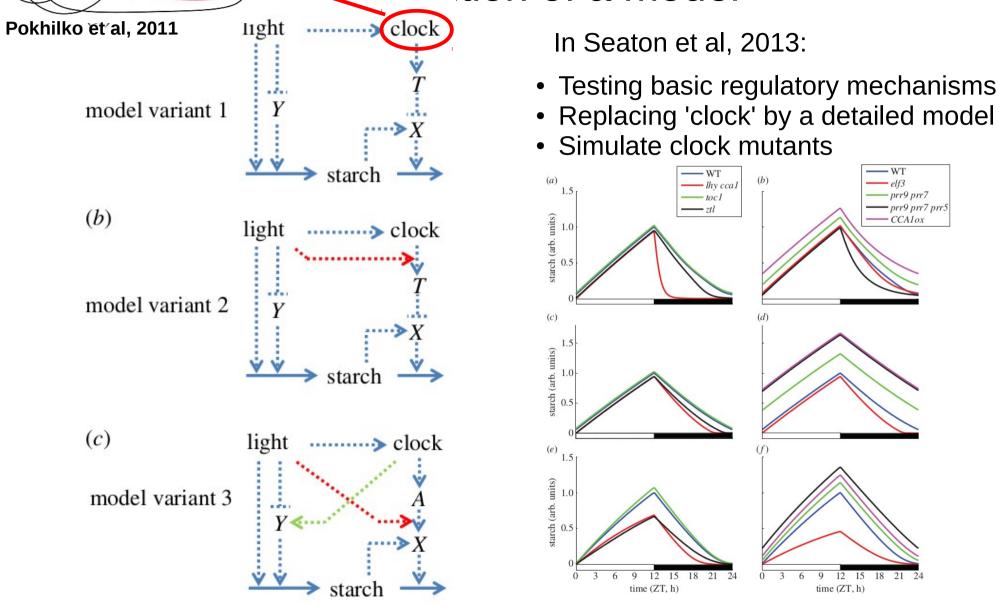


In Seaton et al, 2013:

- Testing basic regulatory mechanisms
- Replacing 'clock' by a detailed model
- Simulate clock mutants



The evolution of a model

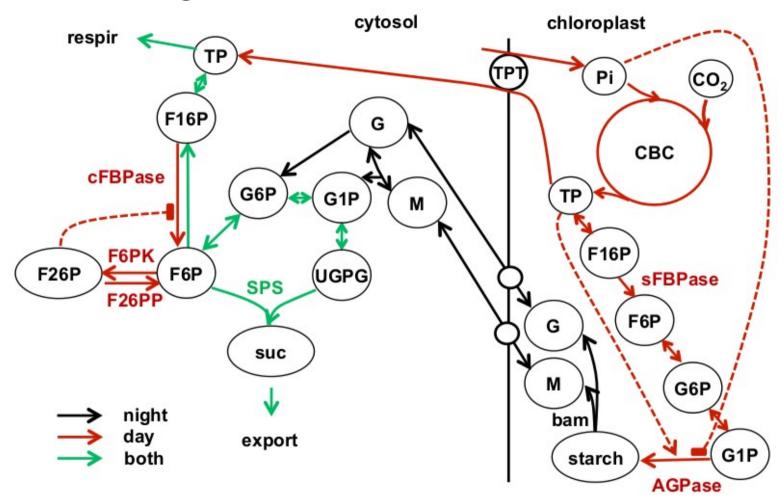


Conclusions: • Variants 2 & 3 ok, more tests needed

TOC1 GI - LUX ELF4 ELF3

• Components A,X,Y remain hypothetical

Adding more details of metabolism



- Carbon fixation
- Starch synthesis
- Starch breakdown
- Sucrose synthesis
- Sucrose export

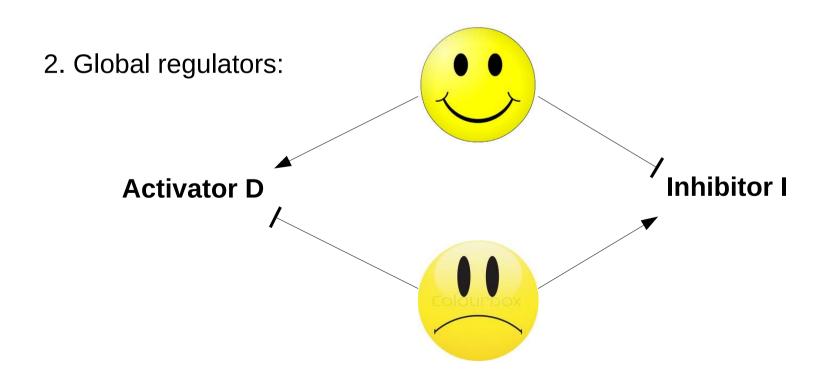
Pokhilko et al, 2014, Mol Biosystems

Include key steps but simplify pathways!

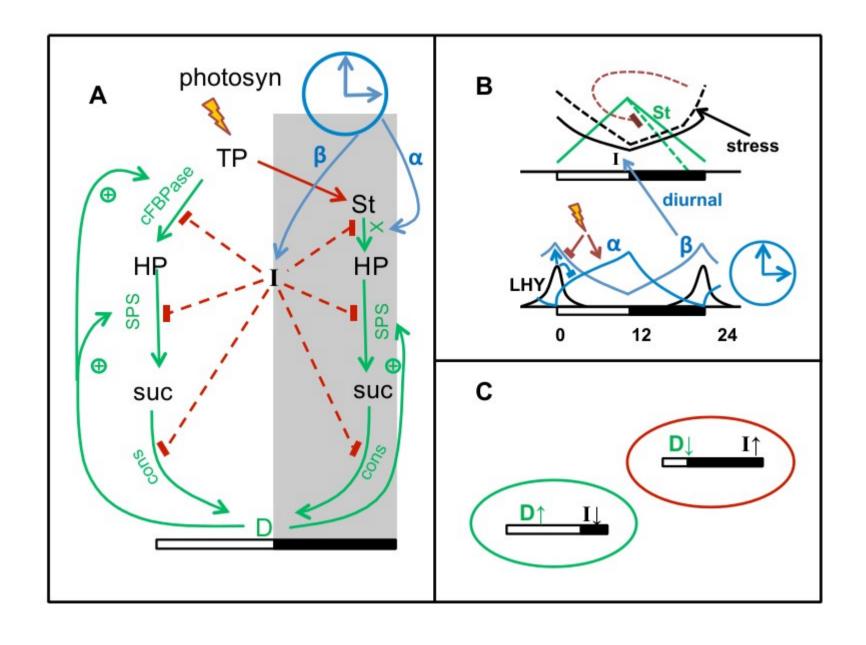
Model assumptions (postulates)

1. Key sensors:

Timer α dark sensor β time-to-dawn carbon limitation

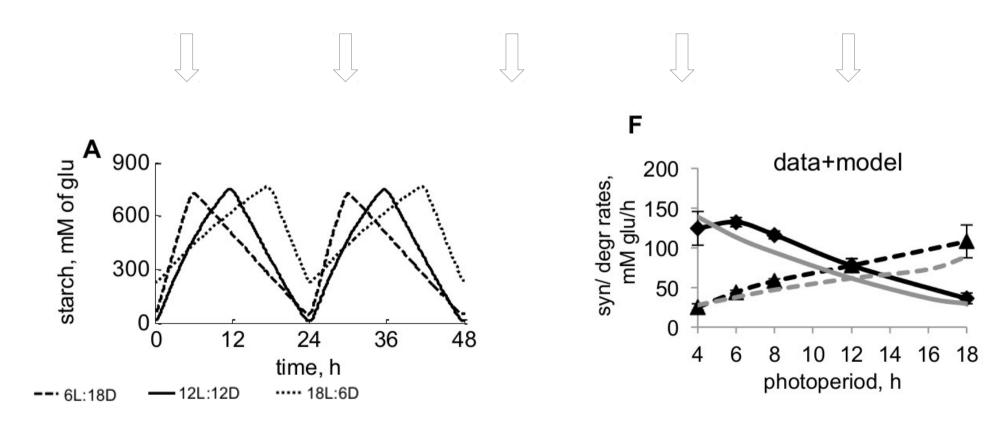


Regulatory principles



Simulations wild-type

Regulatory principles allow to explain wild-type starch turnover under various photoperiods



What are the unknown components?

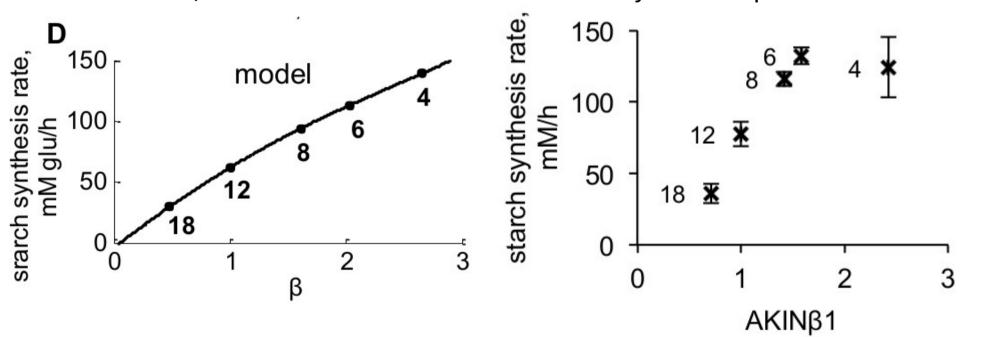
Model allows to make predictions of their behaviour

Helps to identify candidates from expression / proteomics data

For example, the component β :

Predicted peak-levels at dawn

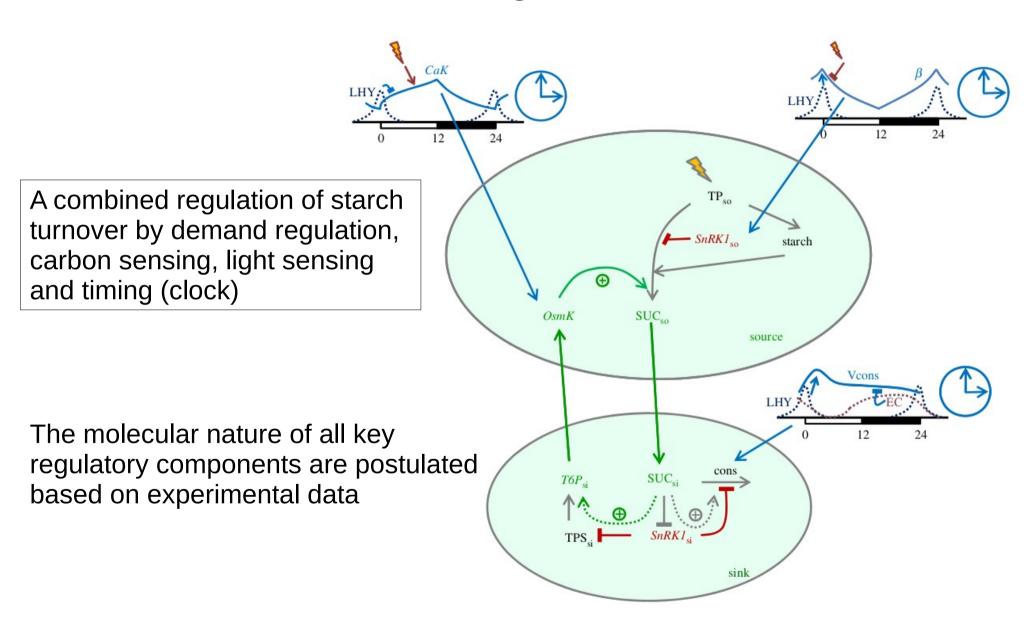
Microarray data for β-subunit of SNRK1



Promotor structure also supports AKIN $\beta1$ as good candidate for β

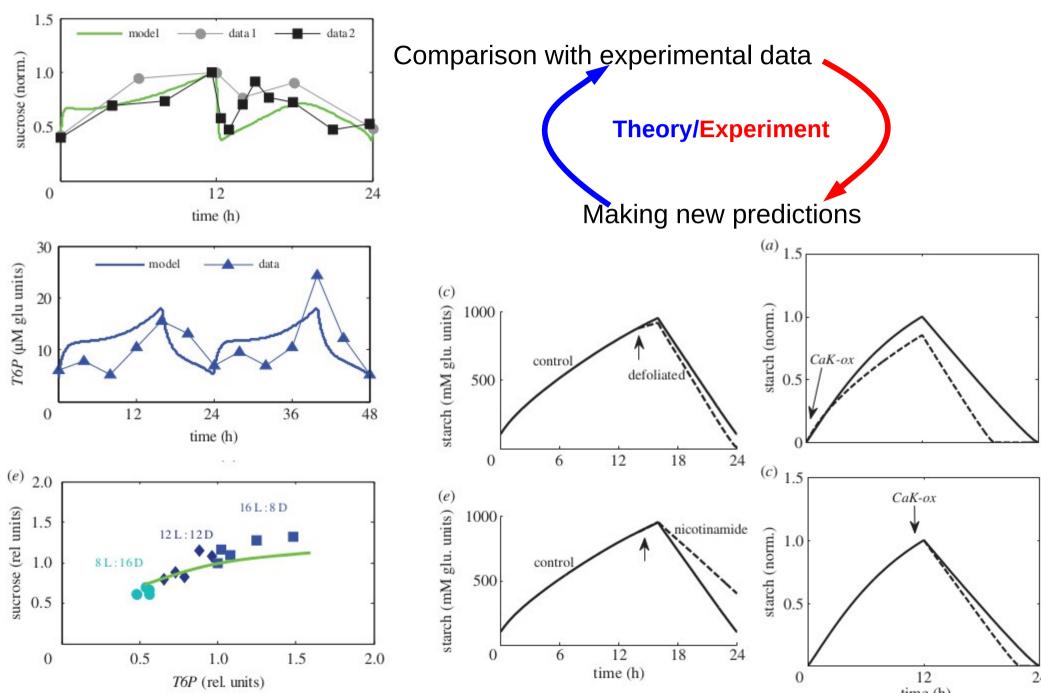
Other regulatory components still unknown!

The third generation



Pokhilko et al, 2015, Roy Soc Interface

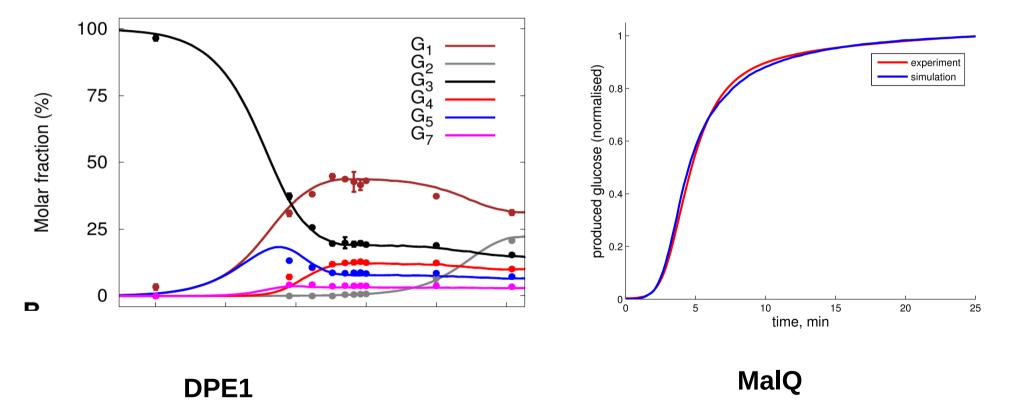
Improved results and new predictions



Outlook – towards designing starch

1. Understand and describe polymer-active enzymes

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1. Understand and describe polymer-active enzymes

OK

Require more data:

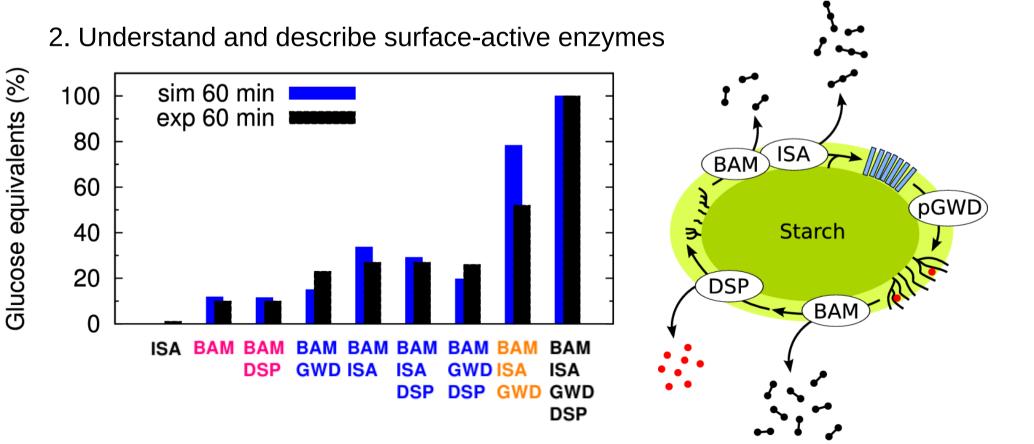
- in vitro kinetics of enzymes
- chain-length distributions for knockouts / synthetic in vitro-systems
- 2. Understand and describe surface-active enzymes

1. Understand and describe polymer-active enzymes

OK

Require more data:

- in vitro kinetics of enzymes
- chain-length distributions for knockouts / synthetic in vitro-systems



1. Understand and describe polymer-active enzymes

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- 2. Understand and describe surface-active enzymes

OK

Require more data:

- in vitro kinetics of enzymes (difficult!)
- synthetic in-vitro systems with crystallised (ideal) starch
- time-resolved data!
- 3. Find the missing links!

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Require more data:

- in vitro kinetics of enzymes (difficult!)
- synthetic in-vitro systems with crystallised (ideal) starch
- time-resolved data!
- 3. Find the missing links!

For example:

- formation of double helices (α-1,4-glucans)
- cooperation of biochemical and biophysical processes

Modelling 3D structure of polysaccharides

POLYS 2.0: An Open Source Software Package for Building Three-Dimensional Structures of Polysaccharides

Søren B. Engelsen, Peter I. Hansen, Serge Pérez²

Received 23 June 2013; revised 18 November 2013; accepted 19 November 2013 Published online 30 November 2013 in Wiley Online Library (wileyonlinelibrary.com). DOI 10.1002/bip.22449

¹ Spectroscopy & Chemometrics, Faculty of Science, University of Copenhagen, Rolighedsvej 30, DK-1958 Frederiksberg C, Copenhagen, Denmark

² Centre de Recherches sur les Macromolécules Végétales, CNRS, BP 53 X, 380451 Grenoble, Cedex, France

The next steps...

- Systematic in vitro characterisation of surface-active and polymer-active enzymes (Rob Field, JIC Norwich)
- Systematic experiments in yeast and combination of enzymes in vitro (Sam Zeeman, ETH Zurich)
- Combine existing modelling approaches (Oliver Ebenhöh, HHU Düsseldorf)



ERA-CAPS Project **DesignStarch**

Postdoc needed!

- Envisaged start: June 2015
- Goals:
 - synthesise starch in vitro and in yeast
 - model these processes
 - predict physico-chemical properties from biochemistry/biophysics
 - design starch!

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